Two Dimensional Series Based Geoelectric Inversion Using Optimized Weights

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ABSTRACT

The paper deals with the problem of joint inversion of Vertical Electric Sounding data sets over two dimensional geologic structures. The aim of the study is to apply a relatively simple forward problem calculation combined with a new joint inversion algorithm. The studied structure consists of three homogeneous resistivity layers. The first layer-boundary surface is horizontal, but the other one is laterally varying. Two simulated data sets were produced, one in strike direction and the other in dip direction. The exact sounding data were calculated by Spitzer's method. Normally distributed random error values of two different standard deviations were added to both exact data sets in order to simulate real measured data. The L_2 norm technique was applied to inversion of the data, in which the model parameters were approximated by series expansion in the forward modeling problem. Due to different standard deviations of the data sets, objective functions of each data set are weighted individually and added in order to produce the joint objective function. In real circumstances these weights are generally unknown. To overcome this difficulty, the so-called the method of optimized weights was applied. The essence of it is that the joint objective function is optimized with respect to both the model parameters and the data standard deviations. During the inversion the second layer-boundary function was determined. For the comparison, individual and joint inversion supported by the method of optimized weights was applied on the two data sets. The best results were obtained by the method of optimized weights.

INTRODUCTION

In the inversion of resistivity data measured over 2D or 3D geological structures, non-uniqueness and ambiguity problems often occur. By using the series expansion method (Gyulai et al. 2007) and applying finite differences (FD) methods to solve the direct problem in the inversion (Gyulai, Ormos and Dobróka 2010), quality of the inversion results can be improved. When different data sets are available for the same measuring site, their integration into a common inversion procedure can also improve the quality of results. In case of joint inversion of two data sets (measured on different physical basis or in various measurement arrays) a more realistic result can be achieved by using weights for the individual data sets. Nevertheless, there are occasional methods to calculate these weights (Julià et al. 2000; Kis 2002; Mota and Monteiro 2006), but there is no general rule to determine them. Treitel and Lines found this as a basic problem in geophysical joint inversion (Treitel and Lines 1999). In this paper the series expansion method is combined with the optimal weighting method (Drahos 2008; Ormos et al. 2008) for two sets of simulated erroneous Schlumberger sounding data over a two dimensional geological structure consisting of three homogeneous resistivity layers.

THE APPLIED METHOD OF JOINT INVERSION

In this paper, the method of optimized weights (Drahos 2008) is applied for the joint inversion of different geoelectric data sets. Let us consider two different types of geophysical measurements. The sets of data and the corresponding sets of theoretical functions relating to the model are denoted by the vectors d_1, $g_1(m)$ and d_2, $g_2(m)$ respectively. The dimensions of the vectors are n_1 and n_2. The unknown model parameters to be determined are the components of the vector m. The following scheme describes the connection between the measured and the theoretical quantities:

$$\mathbf{d_1} = \mathbf{g_1}(\mathbf{m}) + \mathbf{e_1} \tag{1}$$

and

$$\mathbf{d_2} = \mathbf{g_2}(\mathbf{m}) + \mathbf{e_2}\,. \tag{2}$$

The vectors e_1 and e_2 represent the noise, which are independent random numbers. It is assumed that they show normal distribution of zero expected values. The standard deviation is σ_1 for the e_1's and σ_2 for the e_2's. It is important to note that these two standard deviations are also unknown. If the maximum likelihood principle (Menke 1989) is applied for the two data sets separately, the likelihood functions l_1 and l_2 should be maximized with respect to m. Supposing uncorrelated normal distribution of the errors with σ_1 and σ_2 standard deviations for the two data sets respectively, the mathematical form of the likelihood functions are:

$$l_1 = f_1(d_{1,1},\mathbf{m})\, f_1(d_{1,2},\mathbf{m}) \ldots f_1(d_{1,\mathrm{n}_1},\mathbf{m}) \tag{3}$$

where

$$f_1(d_{1,\mathrm{i}},\mathbf{m}) = \frac{1}{\sqrt{2\pi}\,\sigma_1} \exp\left(-\frac{1}{2\sigma_1^2} \left(d_{1,\mathrm{i}} - g_{1,\mathrm{i}}(\mathbf{m})\right)^2 \right), \tag{4}$$

(Menke 1989) and similarly:

$$l_2 = f_2(d_{2,1},\mathbf{m})\, f_2(d_{2,2},\mathbf{m}) \ldots f_2(d_{2,\mathrm{n}_2},\mathbf{m}) \tag{5}$$

where

$$f_2(d_{2,\mathrm{j}},\mathbf{m}) = \frac{1}{\sqrt{2\pi}\,\sigma_2} \exp\left(-\frac{1}{2\sigma_2^2} \left(d_{2,\mathrm{j}} - g_{2,\mathrm{j}}(\mathbf{m})\right)^2 \right), \tag{6}$$

By maximizing l_1 and l_2, two independent m_1 and m_2 estimates are determined respectively. The aim of joint inversion is that one common estimate of m is to be determined. Therefore the product $l = l_1 l_2$ should be maximized. The complete form of l is:

$$l = \frac{1}{(\sqrt{2\pi})^{n_1+n_2}\,\sigma_1^{n_1}\,\sigma_2^{n_2}} \exp\left\{ -\frac{1}{2\sigma_1^2} \sum_{\mathrm{i}=1}^{n_1} \left(d_{1,\mathrm{i}} - g_{1,\mathrm{i}}(\mathbf{m})\right)^2 - \frac{1}{2\sigma_2^2} \sum_{\mathrm{j}=1}^{n_2} \left(d_{2,\mathrm{j}} - g_{2,\mathrm{j}}(\mathbf{m})\right)^2 \right\} \tag{7}$$

The usual way to solve the problem is to minimize the negative logarithm λ of function l:

$$\lambda = -\ln(l) \tag{8}$$

that is

$$\lambda = \frac{n_1 + n_2}{2}\ln(2\pi) + n_1 \ln\sigma_1 + n_2 \ln\sigma_2 + \frac{1}{2\sigma_1^2}\sum_{\mathrm{i}=1}^{n_1}\left(d_{1,\mathrm{i}} - g_{1,\mathrm{i}}(\mathbf{m})\right)^2 + \frac{1}{2\sigma_2^2}\sum_{\mathrm{j}=1}^{n_2}\left(d_{2,\mathrm{j}} - g_{2,\mathrm{j}}(\mathbf{m})\right)^2 \tag{9}$$

If the standard deviations σ_1 and σ_2 are known, the minimization relates only to the rightmost two members of equation (9) and in this case the weights are:

$$w_1 = \frac{1}{2\sigma_1^2} \text{ and } w_2 = \frac{1}{2\sigma_2^2}\ . \tag{10}$$

Because the standard deviations are generally unknown, according to the maximum likelihood principle the minimization must also be done with respect to σ_1 and σ_2. The necessary conditions are:

$$\frac{\partial\lambda}{\partial\sigma_1} = 0 \text{ and } \frac{\partial\lambda}{\partial\sigma_2} = 0\,. \tag{11}$$

If these conditions are fulfilled, one gets:

$$\sigma_1^2 = \frac{1}{n_1}\sum_{\mathrm{i}=1}^{n_1}\left(d_{1,\mathrm{i}} - g_{1,\mathrm{i}}(\mathbf{m})\right)^2 \text{, and } \sigma_2^2 = \frac{1}{n_2}\sum_{\mathrm{j}=1}^{n_2}\left(d_{2,\mathrm{j}} - g_{2,\mathrm{j}}(\mathbf{m})\right)^2\ . \tag{12}$$

On inserting σ_1 and σ_2 from equations (12) into the expression of λ in equation (9) and neglecting its constant part, the joint objective function will have the form:

$$\lambda = \frac{n_1}{2}\ln\left(\frac{1}{n_1}\sum_{\mathrm{i}=1}^{n_1}\left(d_{1,\mathrm{i}} - g_{1,\mathrm{i}}(\mathbf{m})\right)^2\right) + \frac{n_2}{2}\ln\left(\frac{1}{n_2}\sum_{\mathrm{j}=1}^{n_2}\left(d_{2,\mathrm{j}} - g_{2,\mathrm{j}}(\mathbf{m})\right)^2\right). \tag{13}$$

This function does not contain σ_1 and σ_2. The next step is to find the minimum of λ, this being a function of the model parameters m. Once this minimum is found, it is the minimum with respect to σ_1 and σ_2 too, because of the conditions in equation (11). Then the estimates of σ_1

and σ_2 can be calculated from equations (12). Their only further role is in the calculation of the standard deviations $\sigma(m_i)$, which measure the uncertainty of the model parameter estimates. Drahos (Drahos 2008) applies this method to the joint inversion of simulated gravity-magnetic, well logging and refraction data sets.

APPLICATION TO VES OVER TWO DIMENSIONAL STRUCTURES

The studied two dimensional model contains three layers with homogeneous resistivities, which are: ρ_1=10 ohmm, ρ_2=5 ohmm and ρ_3=20 ohmm. The first layer thickness is h_1=40 m and the h_2 layer thickness is laterally varying. The structure of the model is shown on figs. 1a-d where the layer boundaries are indicated with a white line. In the inversion the resistivities and the h_1 are supposed to be known. The exact error free solution for the model was calculated by Spitzer's FD numerical code (Spitzer 1995) so, that data set 1 contains 29 measurement lines parallel with the strike direction and in each line there are 19 Schlumberger array VES data. Data set 2 contains 29 measurement lines in dip direction containing 19 Schlumberger array VES data. In order to simulate the erroneous measured data the exact error free solution values were contaminated by 2% and 4% Gaussian noise (which corresponds to σ_1=0.00860 and σ_2=0.01703 on the logarithmic scale) producing the "measured" $\log\rho_{a1,i}^{(M)}$ and $\log\rho_{a2,j}^{(M)}$. The appropriate objective function is:

$$\lambda = \frac{n_1}{2}\ln\left(\frac{1}{n_1}\sum_{strike}\sum_{i=1}^{n_1}\left(\log\rho_{a1,i}^{(\mathrm{M})} - \log\rho_{a1,i}^{(\mathrm{T})}(\mathbf{m})\right)^2\right) + \frac{n_2}{2}\ln\left(\frac{1}{n_2}\sum_{dip}\sum_{j=1}^{n_2}\left(\log\rho_{a2,j}^{(\mathrm{M})} - \log\rho_{a2,j}^{(\mathrm{T})}(\mathbf{m})\right)^2\right) \quad (14)$$

The optimum weight method is combined with the series expansion inversion method (Kis 2002; Prácser 2004; Dobróka and Szabó 2004; Ormos and Daragó 2005; Gyulai et al. 2007; Dobróka et al. 2009; Gyulai, Ormos and Dobróka 2010). In the series expansion inversion method continuous lateral variation of the petrophysical and geometrical parameters are supposed. These laterally varying model parameter functions are approximated by series expansion functions which interpolate among the sounding points. In the papers (Kis 2002) and (Prácser 2004) Chebyshev polynomials, Fourier series and spline approximation were applied. Regardless that the model is two dimensional, at each sounding point one dimensional model is applied. In our case the approximation function for the $h_2(x)$ layer boundary function is a Fourier series:

$$h_2(x) = \sum_{q=-Q}^{Q} B_q \Phi_q(x), \tag{15}$$

where $\Phi_q(x)$'s are the sine and cosine basis functions and the B_q's are the expansion coefficients respectively. In the series expansion 23 coefficients are used. The problem of the optimum number of coefficients is studied by Kavanda et al. (Kavanda et al. 2006). The vector of the model parameters for the three layer model is:

$$\mathbf{m} = (h_1, B_{-11}, .., B_0, .., B_{11} \ , \ \rho_1, \rho_2, \rho_3). \tag{16}$$

By minimizing λ in equation 14 the B_q coefficients are determined, and the $h_2(x)$ layer boundary function can be calculated for arbitrary *x* arguments (equation 15). Four inversions were done:

case *a*: separate inversion of data set 1,

case *b*: separate inversion of data set 2,

case *c*: joint inversion with equal weights,

case *d*: joint inversion with automatically optimized weights according to equation 14.

The qualities of the results are characterized by the data- and model distance. The data distance is the standard deviation of the logarithmic data difference on linear scale given in percents. The model distance is the standard deviation of the difference of the known exact and the estimated $h_2(x)$ values in percents (Dobróka et al. 1991). In table 1 data distances approximate the input errors within 5-7% accuracy.

case	data set	data distance 1	data distance 2	model distance
a	data set 1	3.74 %		9.40 %
b	data set 2		1.86 %	5.50 %
c	data set 1+2	3.76 %	1.92 %	7.45 %
d	data set 1+2 opt.	3.77 %	1.86 %	3.63 %

Table 1: Inversion results of simulated erroneous VES data over a two dimensional structure. In the table there are the separate inversion results (cases a, b), the usual joint inversion results with equal weights (case c) and the optimized weight method results (case d).

By comparing cases *a* and *b*, the model distance for case *a* is greater than for case *b*. For case *c* the model distance is in between for cases *a* and *b*. The best result is found for case *d* in which the model distance is the least one.

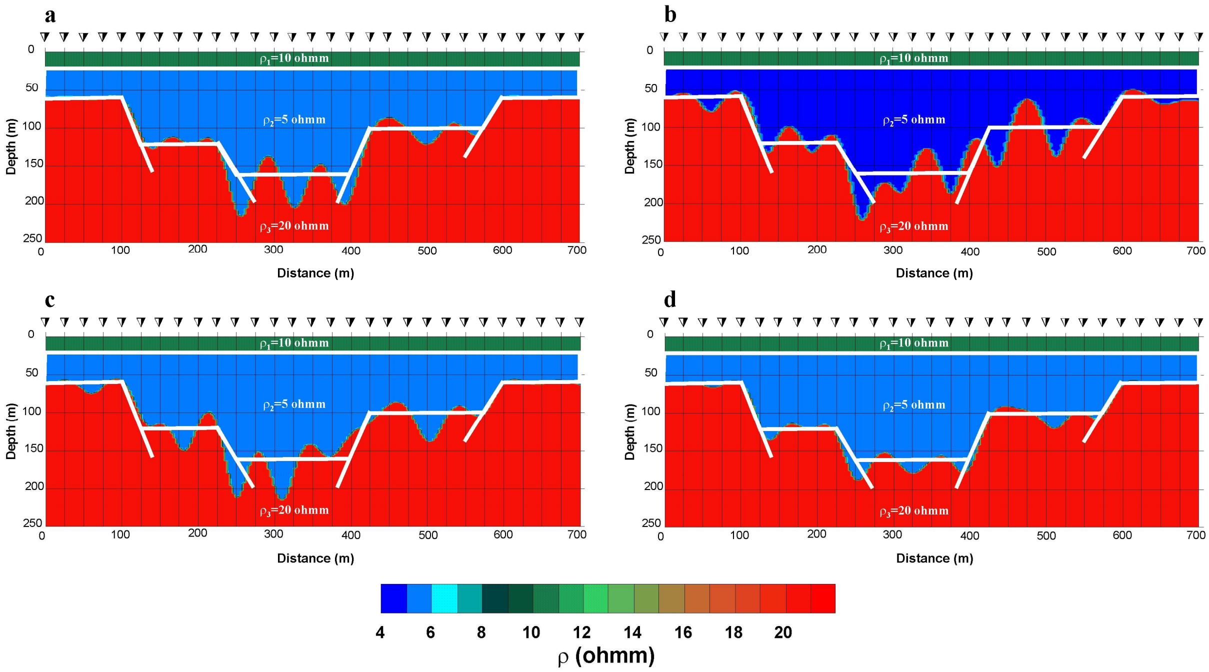

Figure 1: Graphical representation of the inversion results of simulated erroneous VES data over a two dimensional structure. The graphs show the approximations of the exact h_2 layer with the estimated curved boundary functions for cases a, b, c and d. Best approximation is seen on graph d.

On figure 1 a – d the cross section of the model is shown. The estimated $h_2(x)$ boundary function approximates the exact error free boundary with different degree. It can be seen, that the best approximation was found for case *d*. These results show that the joint inversion procedure with optimized (automated) weights gives the most accurate result in comparison with that of the equal weights solution and the separate inversions.

CONCLUSION

In the optimized weight method the complete form of the Maximum Likelihood function is taken into consideration. The essence of the method is that the joint objective function optimization relates also to the standard deviations, and it can be done analytically. As a result the modified form of the joint objective function (equation 13) does not include data error standard deviations. The minimization the objective function with respect to the model parameters means automatic optimization according also to the data standard deviations. The method can be regarded as the generalization of the L_2 norm inversion technique. The expression of the joint objective function can easily be adopted into existing L_2 norm algorithm. The applicability of the series expansion inversion method of electrical soundings measured along a profile for inhomogeneous environments was proved on several examples (Gyulai and Ormos 1999, Gyulai et al. 2007). Its combination with the method of optimized weights produced the best result when compared with the separate and equal weights solutions.

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