# SOLUTION OF A REFRACTION INVERSE PROBLEM USING FIELD DATA MEASURED IN THE TELKIBÁNYA REGION

***Anikó Noémi Paripás***
*University of Miskolc, Department of Geophysics, noemiparipas@freemail.hu*

## 1. INTRODUCTION

In order to be able to determine the boundary between two layers of different rock types, a reliable observation and data process is needed. Seismic refraction measurement is an adequate method for separating the riverside sediments from the volcanic rocks and differentiating the volcanic rock types found in the Telkibánya region from each other: the Sarmatian rhyolite, the Sarmatian dacite and the Sarmatian andesite. For the interpretation of measured data a reliable inversion technique has to be used. For this purpose an inversion technique was developed in the Department of Geophysics, University of Miskolc. To know the reliability of such an interpretation method, testing has to be done on different types of geological structures with quasi measured correct traveltime data.

The used inversion method was developed for multilayered cases based on the fact that lateral changes in the physical parameters (e.g. layer thicknesses, propagation velocities and resistivities) are described by adequately chosen continuous basic functions expanded in series. Coefficients of the functions are estimated with a qualified linearized LSQ and other types of inversion. This approximation was successfully applied in inversion and joint inversion of geoelectric and seismic data [1] [2] [3] [4] [5].

## 2. THE INVERSION METHOD

Knowing the geological structures found in nature an inversion method has to be developed that is capable of treating slow lateral changes both in thicknesses and velocities [4]. This method was applied successfully for refracted data. With the help of this method determination of refracting horizons and layer velocities are possible. Lateral changes in both wave propagation velocities in the media and layer thicknesses are described by series expansions using adequately chosen basic functions. The $p_i(x)$ geometrical and physical parameters of the investigated model (e.g. thicknesses and velocities) can be defined as follows (1).

$$p_i(x) = \sum_{j=1}^{Ji} C_{ij} \cdot F_{ij}(x) \tag{1}$$

In the equation $i$ represents the layer numbers, $J_i$ the number of functions defining the $i^{th}$ parameter, $F_{ij}(x)$ the $j^{th}$ basic function of the $i^{th}$ parameter and $C_{ij}$ the $j^{th}$ expansion coefficient of the $i^{th}$ parameter. The basic function $F_{ij}(x)$ in case of trigonometric series takes the form of $sin(j*2\pi x_i/X)$ and $cos(j*2\pi x_i/X)$ ($X$ means here the length of the profile). $C_{ij}$-s do not depend on the distance $x$ along the profile and therefore they are suitable for determination (estimation) by inversion.

In solving the direct problem some assumptions and approximations have to be taken. Wavelength determined by the highest dominant frequencies is attainable in investigation of near-surface velocity conditions. As it was proved in case of geoelectric application it is the trigonometric function series that is really suitable for describing geological structures and the number of coefficients as well [7].

In the solution of the inverse problem the estimation of the $C_{ij}$ coefficients has to be carried out. After solving the inverse problem it is necessary to calculate the physical parameters of the 2D model from the estimated $C_{ij}$ coefficients along the profile. The coefficients of the functions are calculated by a qualified linearized LSQ inversion, according to which the nonlinear problem can only be solved iteratively. The described inverse problem is formulated in the following form (2):

$$\vec{c} = \left(\underline{\underline{G}}^T \underline{\underline{G}} + \lambda \underline{\underline{I}}\right)^{-1} * \underline{\underline{G}}^T \vec{t} \qquad (2)$$

where $\vec{c}$ means the correction vector of the coefficients (compared to the previous estimation), $\vec{t}$ the vector of the differences between the observed and (from the estimated coefficients) calculated first arrival times, and $\underline{\underline{G}}$ the (Jacobi-) matrix of the partial derivatives of the traveltimes according to the $C_{ij}$ function coefficients. $\underline{\underline{I}}$ represents the unit matrix, and the scalar $\lambda$ is the damping factor.

The aimed investigations are qualified with the help of the following standards. The relative deviation $D$ in the data space is defined as follows (3):

$$D = \sqrt{\frac{1}{I} \sum_{i=1}^{I} \left( \frac{T_i^{(observed)} - T_i^{(calculated)}}{T_i^{(calculated)}} \right)^2} \qquad (3)$$

(where $i$ = 1,…, $I$ denotes the number of time data).
The relative deviation in the model space can be calculated from the values of $p_n(x)$ physical model parameters at $m$ seismic source points in the following way:

$$d = \sqrt{\frac{1}{M} \frac{1}{(2N-1)} \sum_{m=1}^{M} \sum_{n=1}^{2N-1} \left( \frac{p_n(x_m)^{(exact)} - p_n(x_m)^{(estimated)}}{p_n(x_m)^{(exact)}} \right)^2} \qquad (4)$$

(where $m$ = 1,…, $M$ denotes the number of source points, $x_m$ the distance of the $m^{th}$ source point along the profile, $n$ = 1,…, 2$N$-1 the number of model parameters). In case of field measurements the relative deviation between the estimated and starting model parameters can be calculated in the similar form as in Eq.4.

## 3. OUTLINES OF THE GEOLOGICAL SITUATION OF THE TELKIBÁNYA REGION

Telkibánya is situated in the Tokaj Mountains, in the north-eastern part of Hungary. These mountains consist of mostly Miocene, Sarmatian volcanic rocks. Older rock types, such as Early Paleozoic or Late Permian rocks can be found only on the surface in the form of minor outcrops in the north-eastern part of the mountains. Mesozoic formations, such as marine sedimentary rocks were identified only in the south-eastern periphery. The first volcanic units were accumulated on the Mesozoic rocks intercalating with marine sediments. Volcanic activity started 17 Million years ago, in the Miocene, Early Badenian with a submarine rhyodacite-ignimbrite flow. The traces of this volcanic activity and the overlying sediments were found near Telkibánya only in boreholes, similar to the following Late Badenian andesitic-dacitic lava flows and subvolcanic bodies [7] [8].

The shallow marine sedimentation continued in the Sarmatian, this way large masses of rhyolitic pyroclasts were accumulated on land. These types of rocks can be found both in the southern and northern parts of the mountain. Ignimbrites and tuffites also appear in the pyroclastic sequence and rhyolitic lavadomes were formed, too. Andesitic lavaflows and subvolcanic bodies are common in the central part of the mountain of which the chemical character varies from basic pyroxene andesite to dacite [7].

In the Late Sarmatian an intense post-volcanic hydrothermal activity took place, according to which several quartz veins and silica bodies remained. Silica bodies were formed in the steam-heated alteration zone. A large number of important industrial minerals (e.g. kaolinite, bentonite, illite, and diatomite) were formed in these hot-spring basins. After the volcanic activities the mountains were fractured by tectonic movements and the peripheral parts gradually dropped. Valleys and basins formed along the major fractures [8].

## 4. INVESTIGATIONS ON SYNTHETIC GEOLOGICAL STRUCTURES

To make sure that the used inversion method is reliable geological structures of synthetic data has to be investigated. The most problematic part of a certain geologic structure is a steep layer boundary even with step-like changes in that. In the tested synthetic 3-layered geologic structure the first half of the section has relatively slow changes it the depth while in the second half a stair shape can be seen. The propagation velocities are constant in each layer: 400 m/s, 1400 m/s and 2400 m/s. The length of the section in x direction is 250 m, its depth is 20 m; 13 sources - with 20 m spacing - and 126 geophones - with 2 m spacing - were used. This investigation was carried out using correct synthetic data computed with FD-Vidale type ray tracing, a method based on FD approximation of the eikonal equation for calculating synthetic traveltimes using the ReflexW software by Sandmeier [9]. With this method transmitted, diffracted and head waves are also taken into account. This way the complexity of the model is not limited at all. Only the first arrivals can be calculated, so the method is only suitable to simulate picked refraction traveltimes [10].

The calculated synthetic data were used in the inversion method as quasi "measured data" without additional error. In this process 17 coefficients (unknowns) were used, number of data was 783. As results of the inversion traveltime data are also got from the estimated coefficients (*Fig. 1*). Here traveltime curves of the initial (brown dots) and the estimated model (dark blue continuous lines) are compared to each other. After the inversion the calculated geological structures can be drawn; the comparison of the initial model and the gained one can be seen in *Fig. 2*. In this figure the layers of the initial model are shown in different colours and the calculated layer boundaries are shown as dark blue continuous lines. As a result the model distance *d* (Eq.4) was 10.7 %, the data variance *D* (Eq.3) was 1.5 %. These mean good fitting between the first half of the initial and calculated structure and slightly good fitting in the second half of the section (*Fig. 2*).

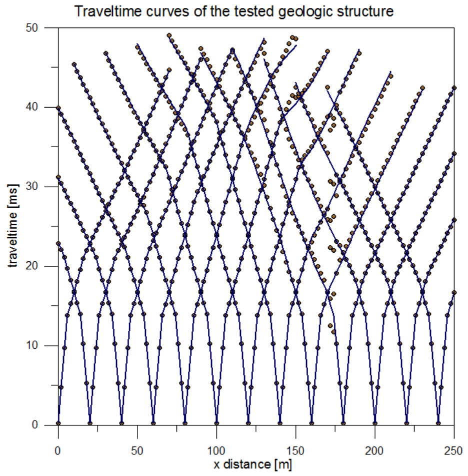


***Figure 1*** *Comparison of the traveltime curves of the initial and calculated geologic structures*

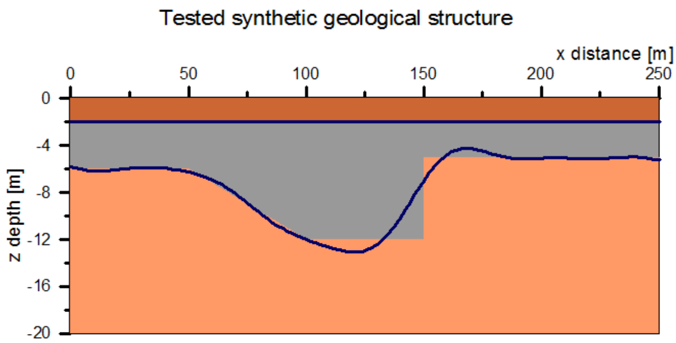


***Figure 2*** *Comparison of initial and calculated geologic structures*

## 5. FIELD INVESTIGATIONS IN THE TELKIBÁNYA REGION

In the investigated territory our aim was to clarify the geological situation. Although the region has been studied for many years, some parts of its geology is not really sure yet. In our investigated area the question was the depth of the stream sediments found on the surface and if it can be distinguished, the shape and place of the layer boundary between the dacite and rhyolite rocks. It was thought to be a great help in mapping the region. Magnetic, geoelectric and seismic refraction measurements were carried out, from which one part of the refraction measurements and results will be shown.

The refraction observation was carried out in a valley near Telkibánya along a small stream. The length of the section of which the interpretation can be seen here (*Fig. 3*) was 94 metres with 48 geophones, with 2 m spacing. The observation was carried out with 17 sources along the section. The boundary between the dacite and the rhyolite was estimated near to the area of our investigation.

In the inversion process Fourier approximation was used with a series of 17 elements for the description of the thickness; 5 elements for the propagation velocity in the first layer and 3 elements for the propagation velocity in the second layer. These allow the lateral change of velocity in each layer and the change of the depth of the layer boundary between the two layers in the structure. In solving the equation (Eq.2) with 25 unknowns 55 iteration steps were done, at the end of which the data variance *D* (Eq.3) was 17.3 % which is not a quite good fitting. This result can point to the high level of inhomogeneity especially compared to the wavelength used in the observation process.

The investigation depth was approximately 4 metres, in which mostly the effects of the surface are observable; the contact of the two volcanic rock types is not really recognizable. However as one can see in *Fig. 3*, the boundary between the riverside sediments and the volcanic rocks is definite, while the colours show that there are changes in velocities in each layer. In the near-surface layer the propagation velocity varies from 200 to 1100 m/s (shown in cold bluish colours), which is a quite big velocity change in one layer; it can refer to a high rate of inhomogeneity. In the other hand the velocity of the lower media varies from 1900 to only 2300 m/s (shown in warm colours from yellow to red); this small change does not allow us to precisely tell if there are one or two different rock types below the upper layer. The velocities in the upper part refer to sandy, gravely sediments, while the velocities in the lower part show volcanic, more compact rocks. The velocity growth at about 60 metres in the section below (*Fig. 3*) can refer to the boundary of the two volcanic rocks; this would mean that their contact is nearly perpendicular to the surface (the perpendicularity was known from geoelectric measurements). The lower velocities (yellowish colours) denote the dacite while the higher velocities (red colour) mark the rhyolite rock.

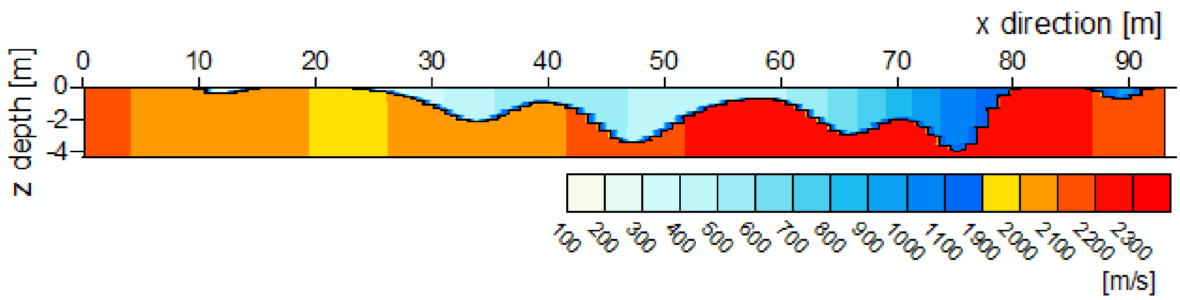


***Figure 3*** *Propagation velocity section of the investigated area in the Telkibánya region*

As it can be seen in *Fig. 3*, the shape of the layer boundary is really curved. The reason of this much curved line can be two things. One reason can be the Fourier approximation, because using it sharp edges are likely to turn into some smooth, wavering lines (as it was seen in *Fig. 2*). However more likely is a reason according to which the high level of inhomogeneity causes the wavering and the geologic structure (mostly the layer boundary) is just like the one seen in *Fig. 3;* with great changes in depth and velocities.

## 6. SUMMARY

The synthetic and field measurements and results proved that the used inversion method is capable of dealing with shallow, elongated structures with relatively slow changes in the layer boundary. The bottom of the riverside sediments was easily separated from the volcanic rocks and according to the propagation velocities the place of the boundary between the dacite and the rhyolite rocks could be also estimated.

## ACKNOWLEDGEMENTS

The author wants to express her thanks to the BSc students in Earth Sciences and Engineering, University of Miskolc for data observation, to the Hungary-Slovakia CBC Programme (Project Code: HUSK 0801/116) and to the Hungarian Scientific Research Fund (OTKA K62416) for the support of the research.

## REFERENCES

[1] DOBRÓKA, M.: **Inversion of guided-wave dispersion data**. P155. EAGE 59th Conference and Technical Exhibition, Geneva, 1997.

[2] GYULAI, Á. and ORMOS, T.: **A New Procedure for the Interpretation of VES Data: 1.5-D Simultaneous Inversion Method.** Journal of Applied Geophysics, Vol. 41. 1999. pp 1-17.

[3] De Nardis, R., Cardarelli, E. and Dobróka, M.: **Quasi-2D Hybrid Joint Inversion of Seismic and Geoelectric Data.** Geophysical Prospecting, Vol. 53. 2005. pp 705-716.

[4] ORMOS, T.: **Inversion of Refracted Travel-times for Near-surface Investigation.** EAGE 64th Conference and Exhibition, Florence, 2002. 4 p.

[5] ORMOS, T. and DARAGÓ, A.: **Parallel Inversion of Refracted Travel Times of P and SH Waves Using a Function Approximation**. Acta Geodaetica et Geophysica Hungarica, Vol. 40. 2005. pp 215-228.

[6] GYULAI, Á., DOBRÓKA, M. and KAVANDA, R.: **The strategy of joint inversion using function series**. EAGE 12$^{th}$ European Meeting of Environmental and Engineering Geophysics, Istanbul, 2007.

[7] GYARMATI, P.: **Intermediate volcanism in the Tokaj Mts.** Annales Inst. Geol. Publ. Hung, 58, 1977, 196 p.

[8] HARTAI, É. (ed.): **Earth Science Field Training Park Telkibánya**. Demonstration Course BSc in Earth Science Engineering "Know the Earth", 2010, 20 p.

[9] VIDALE, J.: **Finite-difference calculation of travel times**. Bulletin of the Seismological Society of America, 78(6), 1988, pp 2060-2076.

[10] SANDMEIER, K. J.: **ReflexW Version 4.0 Windows 9x/NT/2000/XP-program for the processing of seismic, acoustic or electromagnetic reflection and transmission data**. Manual. Karlsruhe, 2006, 400 p.