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2D Series Expansion Based Geoelectric Inversion Using Optimized Weights

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SUMMARY

In the joint inversion of geophysical data measured above 2D or 3D geological structures non-uniqueness and ambiguity problems often occur. An other problem in integrating various data sets in a single inversion algorithm (simultaneous- or joint inversion) is the use of appropriate weights regulating the contribution of the given data set to the solution. In order to reduce non-uniqueness and ambiguity problems in this paper we use series expansion in the discretization of the laterally varying model parameters resulting in a much lower number of unknowns. In the joint inversion of two data sets containing sufficiently different level of noise we apply optimized weights for balancing between the data sets. It is demonstrated in a numerical experiment that the application of optimized weights together with the use of series expansion in discretization results in stable and reliable joint inversion.

1. Joint inversion using optimal weights

It is a serious problem in joint inversion the determination of the correct contribution of the different type measured data. The usual way is to produce the weighted sum of the individual objective functions, but these weights are generally unknown. There are several attempts for the determination of the weight factors. Dobróka et al. (1991) and De Nardis et al. (2005) normalized the difference of the measured data and the theoretical value with the measured one producing equal magnitude for the component objective functions. Julià et. al. (2000) introduced the "influence parameter" p, which is responsible for the correct contribution between the seismic receiver function and the dispersion data sets. Mota-Dos Santos (2006), inverted resistivity and seismic velocity data with weights α and 1- α. Both p and α were determined by occasional experiments. There is no general rule for the determination of the weight factors. In this paper a possible solution method (Drahos, 2007) is applied in the joint inversion of resistivity data.

The objective function of inversion in case of L_2 norm is written in the form of:

$$\lambda = \sum_{i=1}^{n} \left(d_i - G_i(\mathbf{m}) \right)^2 , \tag{1}$$

where d_i and $G_i(\mathbf{m})$ are the measured data and the corresponding direct problem (theoretical function) respectively, n is the number of the measured data. By minimizing the function λ the estimates of the model parameters **m** are determined. In joint inversion the objective function λ for two different data sets is written in the following form:

$$\lambda = w_1 \sum_{i=1}^{n_1} \left(d_{1,i} - G_{1,i}(\mathbf{m}) \right)^2 + w_2 \sum_{j=1}^{n_2} \left(d_{2,j} - G_{2,j}(\mathbf{m}) \right)^2 , \tag{2}$$

In equation (2) $d_{1,i}$ and $d_{2,j}$ are the data, $G_{1,i}(\mathbf{m})$ and $G_{2,j}(\mathbf{m})$ are the corresponding direct problems respectively. The constants w_1 and w_2 multipliers are the weights which control the contribution of each individual objective functions. The estimates of **m** depend not only on the individual objective functions but also on the magnitudes of w_1 and w_2 which are generally unknown. To overcome this difficulty (Drahos, 2007) proposed optimized weights.

The L_2 norm objective function can be deduced from the Maximum Likelihood Estimation (MLE) method for normal distribution of the errors. The mathematical form of the likelihood function is:

$$L = \frac{1}{(\sqrt{2\pi})^{n_1+n_2} \sigma_1^{n_1} \sigma_2^{n_2}} \exp\left\{ -\frac{1}{2\sigma_1^2} \sum_{i=1}^{n_1} \left(d_{1,i} - G_{1,i}(\mathbf{m}) \right)^2 - \frac{1}{2\sigma_2^2} \sum_{j=1}^{n_2} \left(d_{2,j} - G_{2,j}(\mathbf{m}) \right)^2 \right\} . \tag{3}$$

Function L depends on the measured data, the model parameters and the standard deviations. By maximizing L, the estimates of the model parameters are determined. The usual way to solve the problem is to minimize the negative logarithm λ of function L:

$$\lambda = -\ln(L) \tag{4}$$

that is

$$\lambda = \frac{n_1 + n_2}{2} \ln(2\pi) + n_1 \ln \sigma_1 + n_2 \ln \sigma_2 + \frac{1}{2\sigma_1^2} \sum_{i=1}^{n_1} \left(d_{1,i} - G_{1,i}(\mathbf{m}) \right)^2 + \frac{1}{2\sigma_2^2} \sum_{j=1}^{n_2} \left(d_{2,j} - G_{2,j}(\mathbf{m}) \right)^2 \tag{5}$$

If the standard deviations σ_1 and σ_2 are known, the minimization relates only to the rightmost two members of equation (5) which corresponds to the minimization of λ in equation (2). It is seen from equations (2) and (5) the true values of the weights are:

$$w_1 = \frac{1}{2\sigma_1^2} \quad \text{and} \quad w_2 = \frac{1}{2\sigma_2^2} . \tag{6}$$

The standard deviations are generally unknown. Therefore according to the MLE principle the minimization must also be done with respect to σ_1 and σ_2. The necessary conditions are:

$$\frac{\partial \lambda}{\partial \sigma_1} = 0 \text{ and } \frac{\partial \lambda}{\partial \sigma_2} = 0. \tag{7}$$

If these conditions are fulfilled, one gets:

$$\sigma_1^2 = \frac{1}{n_1}\sum_{i=1}^{n_1}\left(d_{1,i} - G_{1,i}(\mathbf{m})\right)^2 \quad \text{and} \quad \sigma_2^2 = \frac{1}{n_2}\sum_{j=1}^{n_2}\left(d_{2,j} - G_{2,j}(\mathbf{m})\right)^2. \tag{8}$$

On inserting σ_1 and σ_2 from equations (8) into the expression of λ in equation (5) and neglecting its constant part the logarithmic joint objective function will have the form:

$$\lambda = \frac{n_1}{2}\ln\left(\frac{1}{n_1}\sum_{i=1}^{n_1}\left(d_{1,i} - G_{1,i}(\mathbf{m})\right)^2\right) + \frac{n_2}{2}\ln\left(\frac{1}{n_2}\sum_{j=1}^{n_2}\left(d_{2,j} - G_{2,j}(\mathbf{m})\right)^2\right). \tag{9}$$

This function does not include σ_1 and σ_2. The next step is to find the minimum of λ, this being a function of the model parameters **m**. Once this minimum is found, it is the minimum with respect to σ_1 and σ_2 too, because of the constraints of equation (7). Then the estimates of σ_1 and σ_2 can be calculated from equations (8). Their only further role is in the calculation of the standard deviations $\sigma(m_i)$, which measure the uncertainty of the model parameter estimates.

2. Discretization and forward modeling

It is known, that in the inversion of the geophysical data measured above 2D or 3D geological structures the solutions is usually non-unique. The physical answer for non-uniqueness is the joint inversion, though the problem is also related to the discretization method which is an essential step in constructing efficient and stable inversion algorithm. In one hand there is a need to reach as high resolution as possible, which implies the use of large number of unknowns. On the other hand we have usually "inaccurate, insufficient and inconsistent" noisy data set which makes it possible to determine (uniquely and accurately) only a limited number of unknowns. So, in order to achieve acceptable inversion results it is very important to find appropriate discretization, representing a reasonable balance between resolution and stability. In this paper we use Fourier series expansion in discretizing the laterally varying thicknesses of the 2D geological model studied (Gyulai and Ormos, 1999)

$h_n(x) = \sum_{q=1}^{Q_n} B_q^{(n)}\,\Phi_q(x)$ where $\Phi_q(x)$ are the basis functions (sine and cosine in our case), $B_q^{(n)}$ are unknown expansion coefficients. After this we get the model parameter vector characterizing the geological model as

$$\mathbf{m} = (B_1^{(1)},\dots,B_Q^{(1)},\dots,B_1^{(N-1)},\dots,B_Q^{(N-1)},\rho_1,\dots,\rho_N)$$

Using this parameter vector, the theoretical apparent resistivity data (calculated by a 3D finite difference program of Spitzer (1995)) can be written as $G_i(\mathbf{m}) = \rho_a(\mathbf{m}, AB_i/2)$.

Numerical experiment

In order to demonstrate the use of optimized weights in the joint or simultaneous inversion of different data sets we defined a three layered, layer-wise homogeneous 2D model (white lines in Fig. 1-3) and generated two data sets. **Data set I.** was calculated in 29 measurement lines parallel with the strike direction (containing 19 stations in Schlumberger array). In order to simulate quasi measured data set the theoretical values (calculated by the use of Spitzer's FD method) were contaminated by 4% random noise of Gaussian distribution. **Data set II.** in 29 Schlumberger arrays in dip direction (with center points at the triangle symbols). Each arrays contain 19 measurement stations. The calculated data were contaminated by 2% random noise of Gaussian distribution.

In the independent and joint inversion of the two data sets the objective function in Eq. (9) is minimized with $d_{1,i}$ from Data set I. and $G_{1,i}(\mathbf{m})$ theoretical data calculated along measurement lines in strike direction and $d_{2,i}$ from Data set II. and $G_{2,i}(\mathbf{m})$ theoretical data calculated along measurement lines in dip direction. In the discretization the number of expansion coefficients were 3 and 23 in case of h_1 and h_2 , respectively.

The result of independent inversion of Data set I. is shown in Fig. 1. The quality of the result is characterized by the data- and model distance and the mean estimation error (Dobroka et al., 1991). The result of independent inversion of Data set II. is shown in Fig. 2. It can be seen, that because of the difference in the noise containing the two data sets, Fig. 2. shows much better result in both the model distance and the mean estimation error. If we integrate Data set I. and Data set II. into a single (joint or simultaneous) inversion procedure (based on the minimization of the objective function in Eq. (9) the result shown in Fig. 3. can be found. (During the inversion procedure the optimized weights in Eq. (8) were used.) It can be seen, that the data distances are nearly the same, while both the model distance and the mean estimation error is much lower in joint inversion. These results show that the joint (or simultaneous) inversion procedure with optimized weights gives more reliable and accurate result in comparison with independent inversion.

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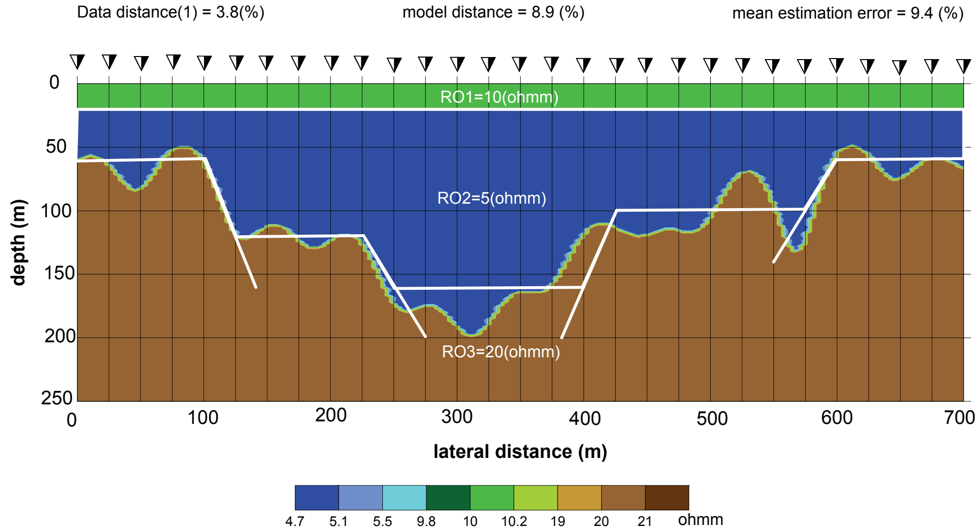

Figure 1. Independent inversion of Data set I.

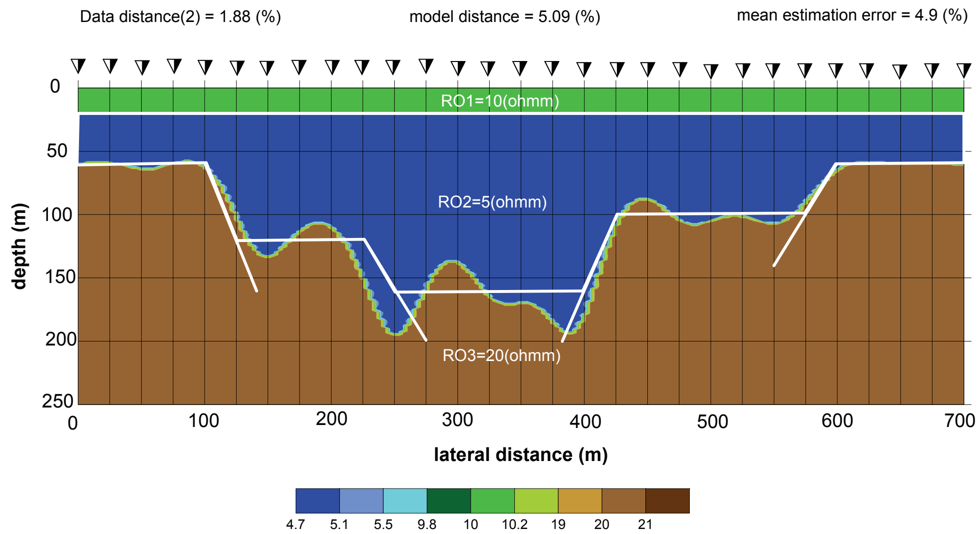

Figure 2. Independent inversion of Data set II.

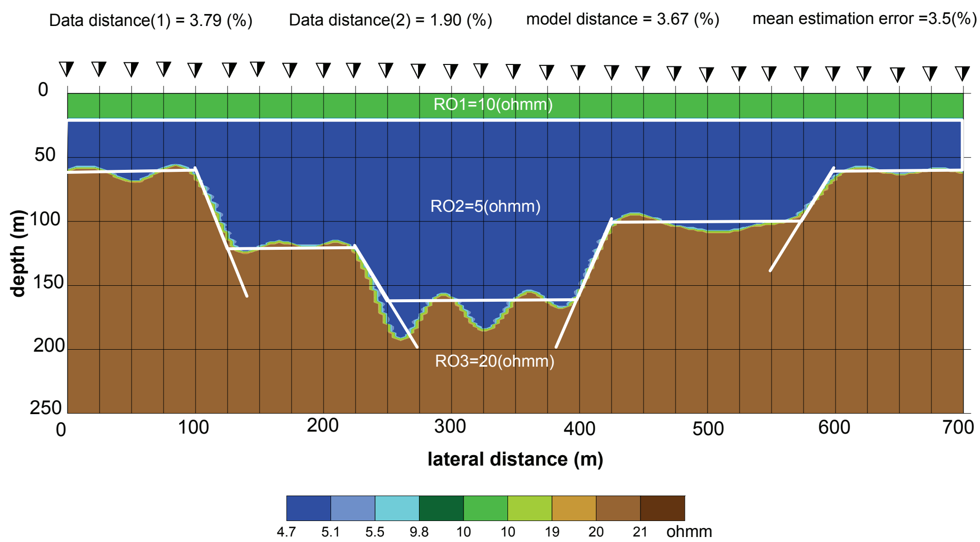

Figure 1. Joint inversion of Data set I. and Data set II.