A quick 2-D geoelectric inversion method using series expansion

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This paper presents the principles of a new inversion method used for determining 2-D geological structures. The basis of the method is that horizontal changes in layer-thicknesses and resistivities of the geological structure are discretized in the form of series expansion. The unknown expansion coefficients are determined by linearized iterative least-squares (LSQ) inversion of data provided by surface geoelectric measurements. The discretization of the 2-D model by means of series expansion gives the possibility to reduce the number of model parameters. Thus, the resulting inverse problem becomes overdetermined and can be solved without the application of additional regularization, e.g., by smoothness constraints, which is usually required for traditional 2-D/3-D inversion. By knowing the expansion coefficients, the local layer parameters are calculated along the profile, point by point. In conformity with the complexity of the model, 1-D forward modeling is applied in the initial iteration steps, then as a continuation the direct problem is handled as a real 2-D problem by means of a finite difference (FD) procedure, i.e., the forward modeling is combined, whereas the unknowns (the expansion coefficients) are the same. This combination of the 1-D and 2-D forward modeling procedures makes it possible to analyze quickly the geological models having considerable lateral variation.

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1. Introduction

For the exploration of 2-D and 3-D geological structures, two different kinds of measurement-interpretation methods are generally used in practice of geoelectric measurements.

In case of models with slow lateral variations, the so-called traditional vertical electric sounding (VES) measurement can be applied well. The local layer parameters characterizing the model are frequently estimated by using apparent resistivity data (sounding curves) determined at relatively sparse stations (50–200 m) with individual 1-D inversion methods (Koefoed, 1979). In order to ensure the fulfillment of a 1-D assumption as much as possible, the measurement electrode arrays are deployed in the strike direction of the geological structure. Then the geological structure is determined from local layer parameters by means of interpolation. In individual inversion, however, equivalence appearing in three or more layered models commonly causes a large uncertainty in parameter estimation, which is less acceptable in practice. This problem can be reduced significantly by applying joint inversion methods (Vozoff and Jupp, 1975; Hering et al., 1995; Haber and Oldenburg, 1997; Misiek et al., 1997; Kis, 1998; Gallardo and Mejú, 2004). In order to give better approximation for geological models with slow lateral variations a new inversion algorithm, the so-called 1.5-D inversion was introduced by Gyulai and Ormos (1999a). By the method all observation data (measured with an electrode array) from each VES station along the survey profile are integrated into one inversion procedure. The lateral change of the model parameters (the dependence of layer resistivities and thicknesses on the lateral coordinates) are approximated by series expansion. In order to assure cost-effective computation the authors use 1-D forward modeling procedure. The efficiency of the 1.5-D method was demonstrated both on synthetic and field data (Gyulai and Ormos, 1997, 1998, 1999a).

In solving 2-D or 3-D inversion problems with the FD forward modeling method (Loke and Barker, 1996; Zhang et al., 2000), piecewise constant resistivities defined on a rectangular grid of cells are assumed. This results in a large number of unknowns normally leading to an underdetermined inverse problem. In order to find a unique solution various additional constraints have to be applied. These constraints may assure the smoothness of the solution (Marquardt-Levenberg method) or even focus on the existence of a boundary (Blaschek et al., 2008). Underdetermined inversion algorithms can efficiently be stabilized with different additional constraints. Using non-physical constraints can have a strong influence on the inversion results.

In order to reduce the non-uniqueness of the problem and to achieve a faster computer code, several authors have combined the use of 2-D and 1-D calculations (e.g., Oldenburg and Ellis 1991; Smith and Booker 1991; Christiansen and Aukén, 2004). Aukén et al. (2005) presented piecewise 1-D laterally constrained inversion procedure for processing and interpreting very large data sets. The locally 1-D models are connected laterally by requiring approximate identity between...
neighboring parameters (typically resistivity and depth) within a specified variance (Aukén and Christiansen, 2004).

In this paper we propose an alternative way to reduce the non-uniqueness inherent in traditional 2-D inversion schemes. We develop further the idea of the 1.5-D inversion presented by Gyulai and Ormos (1999a). For the forward modeling calculations we use the FD algorithm developed by Spitzer (1995). The essential part of the procedure is that we make the parameterization of a 2-D geoelectric earth model in terms of a series expansion of layer-thicknesses and resistivities and define the expansion coefficients as the unknowns of the inverse problem. The use of series expansion results in a sufficient reduction in the number of unknowns compared to traditional FD or Finite Elements Method (FEM) based inversion procedures. This gives the possibility to define the inversion problem as an overdetermined one, without the need of any additional regularization (or smoothness) constraint. In order to reduce the computation time we apply the 1.5-D method (Gyulai and Ormos, 1999a) for the first few iteration steps and use the resultant model as an initial one for the subsequent 2-D inversion. We call the above procedure combined geoelectric inversion (CGI) method. Earlier a similar inversion algorithm was developed to solve the 2-D seismic guided-wave inverse problem by Dobróka (1994) and Dobróka et al. (1995) and a 2-D seismic refraction inverse problem by Bernabini et al. (1988) and by Ormos (2002) and by Ormos and Darágó (2005), in which the lateral variation of the model was discretized by using series expansion and the inverse problem was formulated in terms of the expansion coefficients.

Another application uses the series expansion technique for inverting well-logging, resistivity and IP data (Szabó, 2004; Tauri, 2004; Drahos, 2005; Dobróka and Szabó, 2005; Dobróka et al., 2009).

2. Geoelectric inversion using series expansion

As a starting point, let us make a short overview of the 1.5-D inversion method, because it is also based on the series expansion of the model parameters. In the first phase of the CGI procedure we make use it for finding an initial model for the subsequent 2-D inversion procedure. The linearized CGI algorithm uses the LSQ method for the estimation of the series expansion coefficients, with the help of which thicknesses and resistivities can be derived.

2.1. The 1.5-D geoelectric inversion method

As mentioned in the Introduction, the 1.5-D inversion method was developed for the interpretation of traditional VES measurements (Gyulai and Ormos, 1997, 1998, 1999a). Using this, the lateral variation of a 2-D geological model is described by series expansion using suitably chosen basis functions. The inversion algorithm is an iterative method supported by 1-D forward modeling procedure. The expansion coefficients serve as the unknowns of the inverse problem.

As presented in Gyulai and Ormos (1999a), the series expansion coefficients can be jointly determined from the data of all VES stations along the profile. In this approach, the expansion coefficients, which are the same along the entire profile, constitute the coupling between the different 1-D models used in the forward modeling procedure. Our studies of synthetic and field data demonstrate that the 1.5-D inversion procedure gives accurate and reliable parameter estimation and – in spite of 1-D forward modeling – the results are highly acceptable in the field practice (Gyulai and Ormos, 1999a); Gyulai et al., 2000; Gyulai, 2001), too.

Previous investigations (Gyulai and Ormos, 1997, 1998, 1999a; Kis et al., 1998; Ormos et al., 1999) showed several advantages of the 1.5-D inversion method, i.e. speed, accuracy, stability etc. Based on these results, Kis (1998) generalized the 1.5-D inversion method by using other type of basis functions and studied its impact on the resolution of the equivalence problem in detail.

In 1.5-D approximate inversion a series expansion technique is used to describe the lateral change of the model parameters. We assume that the laterally changing model parameters can be approximated with sufficient accuracy by using the orthogonal Fourier series expansion (Gyulai and Ormos, 1999a):

\[
\rho_n(s) = \frac{1}{2} \sum_{k=1}^{Np} d_k \cos k\frac{2\pi s}{L_p} + \sum_{k=1}^{Np} d_k^* \sin k\frac{2\pi s}{L_p},
\]

(1)

where \( n = 1, \ldots, N \)

\[
h_n(s) = \frac{1}{2} \sum_{k=1}^{Np} c_k \cos k\frac{2\pi s}{L_p} + \sum_{k=1}^{Np} c_k^* \sin k\frac{2\pi s}{L_p},
\]

(2)

where \( \rho_n(s) \) is the resistivity function of the \( n \)-th layer, \( h_n(s) \) is the thickness function of the \( n \)-th layer and \( d_k, d_k^*, c_k, c_k^* \) denote the expansion coefficients. \( N \) is the number of layers, and \( s \) is the lateral coordinate along the profile of total length \( S_p \). Maximum values of \( K_n \) and \( L_p \) can be determined on the basis of the VES points as described by Gyulai and Ormos (1999a). The inversion method is suitable for interpreting most types of electrode arrays (e.g. Gradient, Wenner, dipole–dipole, and pole–pole), and can also be considered as a constrained inversion (Aukén, et al., 2005, 2006; Pellerin and Wannamaker, 2005). In this sense, the constraint is the coupling between the various data sets owing to the fact that the expansion coefficients are the same for all of the calculated data.

2.2. The combined geoelectric inversion method

First of all, for getting more accurate results in the case of abruptly varying 2-D geological structures, it might be necessary to apply exact forward modeling in the 2-D inversion. For this reason, we have further developed the 1.5-D inversion method, because the 1-D forward modeling represents an approximation in this case. In the new linearized CGI inversion algorithm an initial model is produced by the 1.5-D inversion method (first phase) meaning that in the very first iteration steps 1-D forward modeling is applied. Then, in the subsequent iterations (second phase) the slower but more accurate 2-D forward modeling FD algorithm (Spitzer, 1995) is used for calculating the theoretical data and the elements of the Jacobian matrix. The FD forward modeling procedure of Spitzer is developed for 3-D calculations. Its use in a 2-D study results in increased computation time. In spite of this fact we still used Spitzer’s procedure in order to ease the programming effort in our later studies when we extend the investigations to 3-D earth models.

In our CGI method the coefficients of the series expansion serve as unknown model parameters (see Fig. 1). When using series expansion, the layer-thicknesses and resistivities are expressed as continuous functions of the lateral coordinates. Ordinary 2-D modeling assumes piecewise constant resistivities which are defined on a rectangular grid of cells. A mapping from the series expansion onto the grid is necessary in each forward calculation. Namely, for each VES station an individual grid is used, which spacing is the same along the entire profile data demonstrate that the 1.5-D inversion procedure – in spite of 1-D forward modeling – the results are highly acceptable in the field practice (Gyulai and Ormos, 1999a); Gyulai et al., 2000; Gyulai, 2001), too.

Previous investigations (Gyulai and Ormos, 1997, 1998, 1999a; Kis et al., 1998; Ormos et al., 1999) showed several advantages of the 1.5-D inversion method, i.e. speed, accuracy, stability etc. Based on these results, Kis (1998) generalized the 1.5-D inversion method by using other type of basis functions and studied its impact on the resolution of the equivalence problem in detail.

In 1.5-D approximate inversion a series expansion technique is used to describe the lateral change of the model parameters. We assume that

advantage of the series expansion technique is that the number of
unknowns is much less than that of the measurement data, which
makes the inverse problem overdetermined. Thus, we can avoid the
use of any additional non-physical constraints.

In order to validate the accuracy of the inversion results various
tools can be used. In the data space, based on the
\[ L^2 \] norm, the normalized data distance \( d \) is defined
\[
d = \sqrt{\frac{1}{I} \sum_{i=1}^{I} \left( \frac{\rho_{\text{observed}}(i) - \rho_{\text{calculated}}(i)}{\rho_{\text{calculated}}(i)} \right)^2} \times 100\% \quad (3.1)
\]
where \( I \) denotes the total number of measured apparent resistivity
data \( \rho_{a,i} \). In case of inversion of synthetic data calculated on an exactly
known model (with model parameters \( m^{(\text{exact})} \)), the relative model
distance is essential to be computed
\[
D = \sqrt{\frac{1}{Q} \sum_{q=1}^{Q} \left( \frac{m_{q}^{(\text{estimated})} - m_{q}^{(\text{exact})}}{m_{q}^{(\text{exact})}} \right)^2} \times 100\% \quad (3.2)
\]
where \( Q \) is the total number of model parameters.

The accuracy of the parameter estimation is often characterized by
means of the variances, which are derived from the diagonal elements
of the parameter covariance matrix (Menke, 1984). We determined
for the case of series expansion the elements of the covariance matrix
at each VES stations
\[
\sigma_{km} = \sigma_{k xm}(\text{at } x = x_m) \quad (4.1)
\]
where \( \sigma_{k} \) denotes the estimation error of the \( k \)th model parameter
(i.e., resistivity or thickness), and \( \sigma_{km} \) the same at the \( m \)th VES station
(\( x = x_m \)). \( K \) is the total number of the \( p_k(x) \) model parameters
(\( k = 1, 2, ..., K \)), and \( M \) is the number of VES stations along the profile
\( m = 1, 2, ..., M \). \( J(k) \) is the number of elements in the basis function in
the series expansion describing the \( k \)th model parameter, \( \Psi_k(x) \) and
\( \Psi_j(x) \) are the \( i \)th and \( j \)th basis functions and \( \text{COV}_{ij} \) the covariance
matrix elements of the estimated expansion coefficients (defined in
Menke, 1984; Salát et al., 1982).

Fig. 1. Algorithm and flow chart of the combined inversion method (CGI).

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In order to give an overall characteristic of the parameter estimation for the whole model, the mean (percentage) estimation error $F$ is introduced as

$$F = \sqrt{\frac{1}{KM}\sum_{k=1}^{K} \sum_{m=1}^{M} \sigma_{km}^2} \times 100\%$$  \hspace{1cm} (4.b)

In order to characterize the degree of correlation among the estimated model parameters, the correlation matrix is extensively used

$$\text{CORR}_{ij} = \frac{\text{COV}_{ij}}{\sqrt{\text{COV}_{ii}\text{COV}_{jj}}}$$

Because of the large number of the matrix elements, it is useful to introduce a single scalar

$$S = \sqrt{\frac{1}{P(P-1)} \sum_{i=1}^{P} \sum_{j=1}^{P} \left(\text{CORR}_{ij} - \delta_{ij}\right)^2}$$ \hspace{1cm} (5)

called the mean spread (Menke, 1984; Salát et al., 1982), as a general characteristic value describing the model correlation ($P$ denotes the total number of the model parameters, $\delta_{ij}$ is the Kronecker delta symbol).

The smaller the values of $D$, $F$ and $S$ characterizing the whole inverted section, the reliable the results of the 1.5-D and CGI inversion are. We

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**Fig. 2.** Finding the optimal number of coefficients. a.)-n.) Results of CGI-s using different number of coefficient regarding to the second layer thickness (white boxes). d.) The optimal coefficient number is 9. (See also in Fig 3a).

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can use only $F$ and $S$ values for characterization of the inversion results in field cases. For investigations using synthetic data we can additionally use the model distance $D$ too. These values can be applied for the determination of the optimal number of coefficients.

### 2.3. Optimal number of coefficients

In case of series expansion based inversion methods it is very important to specify the optimal number of expansion coefficients. They are depending on the geological structure and the noise, but the structure itself can only be revealed after the inversion. The investigations of 1.5-D and CGI inversions (both with synthetic and field data) show that it can be found many resulting models defined by different number of coefficients at the same minimum of the normalized data distance $d$. Therefore, the choosing of the best resulting model should be based on the minimum of mean estimation error $F$ and on the model distance $D$ in the synthetic case. The determination of the optimal number of coefficients can be treated as an optimization procedure. We made it by executing a series of CGI program runs for different number of coefficient.

To show this procedure we used two synthetic models. For the first case we chose a three-layered 2-D model with straight line boundaries (drawn with white lines in Fig. 2) (Gyulai et al., 2007).

In this investigation only the layer-thicknesses were varying laterally in the model. The model parameters of Fig. 2 are $\rho_1 = 10 \, \Omega \, \text{m}$, $\rho_2 = 5 \, \Omega \, \text{m}$, and $\rho_3 = 20 \, \Omega \, \text{m}$. The layer-thicknesses were described with trigonometric series. In case of the first layer the number of coefficients was 3. For the approximation of the second layer-thickness a varying number of coefficients were chosen from 3 to 29 (marked with white boxes in Fig. 2). Models were estimated, separately, for each coefficient number by using the CGI method. The results are shown in Figs. 2 and 3 (Gyulai et al., 2007). It can be seen in Fig. 3a that the normalized data distance $d$ attains its minimum value rapidly and does not change any more as a function of the number of coefficients. It practically means that there exist many resulting models (with different number of coefficients) at the same data fitting value $d$ (green dots and fitting line in Fig. 3a). Both the mean estimation error $F$ and the relative model distance $D$ (red and blue dots and fitting lines in Fig. 3a) reach a minimum at 9 as the optimum number of coefficients. The results at the best fitting can also be seen graphically in Fig. 2.

The correlation between $F$ and $D$ is near to 1 (see Fig. 3b). Therefore, in case of field applications (where the relative model distance $D$ is not possible to be determined) the optimum number of coefficients is to be found at the minimum of $d$ and $F$, simultaneously. We got similar results for a different types of synthetic models.

For the second case a high resistivity “block” placed in a homogeneous half space (white boxes in Fig. 4) was chosen as a model. The description of this type of model with continuous functions expanded in trigonometric series is not favourable. (The model parameters, and the synthetic data used in Fig. 4 are given later in the text). During this investigation all model parameters (thicknesses and resistivities) were described with trigonometric series using 3–3 coefficients. As an exception, only the coefficient number of resistivity in the 2nd layer was changed from 7 to 15 in five steps (white boxes with coefficient number in Fig. 4) in a series of CGI inversions.

The normalized data distance $d$ as a function of coefficient number reached a minimum at 2.1%. This value remained constant for all resulting models having been estimated by more than 7 coefficients. The minimum value of mean estimation error $F$ was 22.4% at 9 coefficients. (In case of this model type we cannot use the relative model distance $D$, because the layer-thicknesses in a homogenous half space outside of the “block-model” were indefinable). The optimal number of coefficients can be found as a result of a “trial and error” optimization procedure, which can be done manually at the minimum of $d$ and $F$, simultaneously. The fast CGI procedure allows us to perform many inversions for trying different coefficient numbers in a relatively short time. The optimal number of coefficients can differ in the presence of different amount and type of noise even if the model is the same.

### 3. Investigations on CGI method with synthetic data

In order to discuss the most important features of the proposed inversion methods, synthetic data sets generated on a model published previously by Loke and Barker (1996) were used. In this model a 2-D rectangular (200 x 1000 m) body with $\rho = 50 \, \Omega \, \text{m}$ was embedded in a medium of 10 $\Omega \, \text{m}$ at a depth of 25 m. The same model was used at the investigations in Fig. 4. Synthetic apparent resistivity data were calculated for Schlumberger electrode configurations at 16 stations with $\text{AB/2} = 3.2$–200 m by means of the FD code of Spitzer (1995). In Fig. 5 the model (with box) and pseudo section of the calculated Schlumberger apparent resistivities are illustrated.

This data set was used as input data of the 1.5-D and CGI methods, which did not contain any noise. The 2-D model to be investigated...
was constructed with three layers with model parameters varying continuously in the lateral direction. Both the layer-thicknesses and layer resistivities were discretized by means of trigonometric series expansion. All layer-thicknesses and the 1st and 3rd resistivities were described with 3–3 coefficients, for the 2nd resistivity with 17.

Fig. 6a shows the result of the 1.5-D inversion after 35 iteration steps. This was the first phase of the inversion procedure (see Fig. 1). (In Fig. 6a and b the number of expansion coefficients $2L_1 + 1$, $2L_2 + 1$, $2K_1 + 1$, $2K_2 + 1$, $2K_3 + 1$ found in Eqs. 1 and 2 are shown. For example, “Number of coefficients: 3, 3, 15, and 3” means that the two thicknesses were described by using 3 expansion coefficients while the layer resistivity functions $\rho_1(s), \rho_2(s), \rho_3(s)$ were approximated by means of 3, 15 and 3 coefficients in the Fourier series expansion, respectively).

The CGI procedure was continued with 2-D forward modeling in the second phase (Fig. 1), of which result after 4 iteration steps is shown in Fig. 6b. For the comparison between the results of 1.5-D and CGI methods, the same numbers of expansion coefficients was used. In comparing the values of $d$, $F$ and $S$, it can be stated that improvements
are significant, because the \( d = 1.7\% \), \( F = 51.6\% \) and \( S = 0.41 \) values of 1.5-D inversion were reduced to \( d = 0.2\% \), \( F = 5.1\% \) and \( S = 0.26 \) in the CGI procedure. The relatively high value of \( d \) in the first phase of CGI (in case of noiseless data) is the consequence of the 1-D forward modeling used in the first part of combined inversion. The shape of the resistivity anomaly approximates almost exactly the rectangular inhomogeneity, and the resistivity values are still close to the actual ones (\( \rho_1 = 10 \Omega \text{m}, \rho_2 = 50 \Omega \text{m}, \) and \( \rho_3 = 10 \Omega \text{m} \)). In the inversion the total number of unknowns (expansion coefficients) was 27.

It is important to see, how the CGI method works in case of noisy data, so we contaminated the previous data set (Fig. 5) with 2\% random noise of Gaussian distribution. As it is shown in Fig. 6c the combined inversion method gives similar results even in the case of noisy data. In this case - in the presence of noise - the optimum number of coefficients of the resistivity function in the second layer was 9. This procedure and the best model are shown in the previous section in Fig 4b. The relative data distance is 2.1\% (nearly the same as the percentage error of the input data set). The mean estimation error (defined in Eq. (4a)) \( F = 22.4\% \) is relatively large, which was caused by the undulation of the resistivity in the 2nd layer. The amount of \( F \) at this type of model is acceptable. The mean spread characterizing the correlated nature of the set of expansion coefficients was \( S = 0.27 \), which shows slightly correlated (i.e. reliable) results. In case of 1-D inversion the value of \( S \) can reach 0.7 in a normal field case.

We found important to compare the accuracy of the 1.5-D approximation in case of a series of geological models with different variability. To do this, six models of similar shape were defined (see Fig. 7). Each model consists of three layers, where the boundary between the second and third layers is composed of straight line segments. The length of the model, i.e. profile distance varies from 70 m to 2800 m in 6 steps, while the vertical size of all models is unchanged.

Thus we had a series of models, which was varying between 2-D (Fig. 7a) and nearly 1-D (Fig. 7f). The synthetic data were computed by using the FD method for the Schlumberger array at 29 stations along the profile with current electrode spacing (AB/2) between 1.6 and 50 m. Gaussian noise of 2% was added to the synthetic data.

During the tests of the 1.5-D method, the Fourier series expansion was used for discretization. The first thickness was described with 3 coefficients, and the second with 29 coefficients. For seeing clearly this effect, the resistivities did not change here laterally; i.e., they were described with one coefficient for each. Thus the numbers of expansion coefficients were chosen as 3, 29, 1, 1, and 1.

As it is shown in Fig. 7a–f, the relative distance between the exact and the estimated models (the model distance D defined in Eq. (3b)) appreciably decreases with increasing model length. As it was expected, the 1.5-D inversion method (containing 1-D forward modeling) gives better estimates for long models (Fig. 7c–f), where 1-D modeling is a proper approximation. However, in the case of models of Fig. 7a–b, the 1-D approximation can be used only with considerable error.

Because of this reason we continued the calculation with 5 CGI iterations for improving the estimated model. The given accuracy for the most rapidly varying model (Fig. 7a) in case of combined inversion is due to the performance of 2-D forward modeling. The result of the CGI inversion regarding to this model can be seen in Fig. 2d too. We got the value of $D = 2.1\%$ for CGI and $D = 11.4\%$ (Fig. 7a) for the 1.5-D inversion. The values of $d$ are practically the same in both cases.

As a comparison, Fig. 8a shows the 2-D model derived by the CGI procedure (see also Fig. 3d), where $d = 1.9\%$, $D = 2.1\%$, $F = 2.3\%$, and $S = 0.26$ were obtained. Using the smoothness constrained inversion software called RES2DINV (Geotomo Software), which is commonly applied in the geoelectric practice, results represented in Fig. 8b–c were obtained. (Here in order to find the best result we used vertical to horizontal flatness filter ratio = 0.25 (Fig. 8b) and 1.0 (Fig. 8c), ratio of maximum number of model blocks to data points = 50. The rms value calculated by the RES2DINV software was 1.71 for both Fig. 8b and c). Data sets in Fig. 8a–c were contaminated with 2% Gaussian noise.

In Fig. 9a similar model to the one represented by Fig. 8 can be seen with this difference that we can see a series of faults in the second layer. Using CGI method $d = 1.9\%$, $D = 5.1\%$, $F = 4.9\%$, and $S = 0.28$ were obtained. (In case of RES2DINV, we used vertical to horizontal flatness filter ratio = 0.25 (Fig. 9b) and 1.0 (Fig. 9c), ratio of maximum number of model blocks to data points = 50. The rms value calculated by the RES2DINV software was 1.64 for Fig. 9b and 1.71 for Fig. 9c). Data sets in Fig. 9a–c were charged with 2% Gaussian noise.

As the above comparison shows the CGI method solving an overdetermined inverse problem (without the use of any non-physical constraints or smoothing) produces sharper boundaries than RES2DINV, which gives a smooth geological model.

The values of former slowly varying model parameters (i.e. resistivity and layer-thicknesses) were changed randomly by 10% in order to form a new model shown in Fig. 10. The input apparent resistivity data for the inversion were calculated by FD modeling by the Spitzer method, which were then contaminated with 2% Gaussian noise. The corresponding theoretical model can be seen in Fig. 10a. In Fig. 10b–c the CGI inversion...
The number of coefficients can be found. The optimal numbers of coefficients were chosen as 29, 17, 7, and 7. Thus, $d = 1.9\%$, $D = 10.3\%$, $F = 11.6\%$ and $S = 0.15\%$ were obtained. It can be seen in Fig. 10b that the faster the variation of the resistivity, the higher the number of coefficients should be used in case of the CGI procedure. But, as it is also seen in Fig. 10c, the unreasonable increase of the number of coefficients will degrade the quality of the inversion results, which can be avoided by choosing the optimal number of coefficients.

4. Case study

The applicability of the proposed 1.5D and the CGI inversion methods was studied in an environmental geophysical problem. The target area was a waste site near to Miskolc, North-east Hungary. Along the profile 198 data were collected in 9 VES stations. In both cases the numbers of expansion coefficients were the same: 9, 9, 9, 9, 1, and 1. As it can be seen in Fig. 11, the 1.5-D method gave an estimate for the model with extremely large mean estimation error ($F = 150\%$) and relatively high internal correlation among the expansion coefficients ($S = 0.46\%$). On the contrary, the CGI method resulted in significantly decreased data distance and mean parameter estimation error ($F = 16.2\%$). Though the number of unknowns was the same in both inversion problems, but the mean spread (defined in Eq. 5) was obtained sufficiently smaller in case of the CGI procedure ($S = 0.27\%$) indicating less correlated (i.e. more reliable) parameter estimation.

The next field case was the investigation of an aquifer situated at Tisza River, East Hungary. The geologic target was an inhomogeneous gravel sequence of strata with a clayey basement. The structure was approximated by a four-layered model, in which both layer-thicknesses and resistivities were allowed to vary rapidly. The optimal number of coefficients for this case 13, 11, and 13 for the thicknesses and 13, 9, 7, and 2 for the resistivities, respectively. Along the profile 312 data were collected in 13 VES stations. In Fig. 12 the 1.5-D and CGI inversion results are compared. It is also seen in the figure that this technique may visualize the outcrop of layers with thin layer-thickness or nearly the same value of resistivities in adjacent layers. This can be seen, for instance, at $s = 1200\,$m. Comparing 1.5-D and CGI inversion methods, the mean parameter estimation error decreased from 29% to 19% and the mean correlation from 0.35 to 0.29.

5. Conclusion

In conclusion, it can be stated that the new combined geoelectric inversion procedure is promising in determining two-dimensional geoelectric structures. Due to the discretization by means of series expansion, the number of unknowns is relatively small in comparison with common inversion schemes, in which each element of the grid represents an unknown model parameter. It was shown, that because of solving an overdetermined inverse problem (without the use of any non-physical constraints or smoothing) synthetic 2-D geologic models can be reconstructed by the CGI method with high accuracy. In order to reduce the computation time, the CGI method consisted of two phase. At first 1.5-D inversion method (using 1-D forward modeling) was applied to calculate an appropriate initial model for the second phase, where then 2-D forward modeling was applied in the form of a linearized inversion procedure.

At a given noise level, it was shown that more complicated models can only be reconstructed with lower accuracy than a simpler one with the same physical parameters (see Figs. 8a and 9a).

Besides, the unreasonable increase of the number of coefficients degrades the quality of the inversion results, which can be avoided by choosing the optimal number of coefficients (see Figs. 2–4 and 10). Better inversion results can only be obtained by using joint inversion methods integrating additional data sets and/or borehole information.

The 2-D CGI method can easily be generalized for evaluating 3-D geological structures. We think that it would be useful to undertake further research on more complicated models for different observation arrays as well as other types of basis functions in the series expansion and test it on several 3-D field models as an approximation method.
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References


