A joint inversion algorithm to process geoelectric and surface wave seismic data. Part I: basic ideas

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Abstract

For the exploration of near-surface structures, seismic and geoelectric methods are often applied. Usually, these two types of method give, independently of each other, a sufficiently exact model of the geological structure. However, sometimes the inversion of the seismic or geoelectric data fails.

These failures can be avoided by combining various methods in one joint inversion which leads to much better parameter estimations of the model than the independent inversions.

A suitable seismic method for exploring near-surface structures is the use of dispersive surface waves: the dispersive characteristics of Rayleigh and Love surface waves depend strongly on the structural and petrophysical (seismic velocities) features of the near-surface underground.

Geoelectric exploration of the structure underground may be carried out with the well-known methods of DC resistivity sounding, such as the Schlumberger, the radial-dipole and the two-electrode arrays.

The joint inversion algorithm is tested by means of synthetic data. It is demonstrated that the geoelectric joint inversion of Schlumberger, radial-dipole and two-electrode sounding data yields more reliable results than the single inversion of a single set of these data. The same holds for the seismic joint inversion of Love and Rayleigh group slowness data. The best inversion result is achieved by performing a joint inversion of both geoelectric and surface-wave data.

The effect of noise on the accuracy of the solution for both Gaussian and non-Gaussian (sparsely distributed large) errors is analysed. After a comparison between least-square (LSQ) and least absolute deviation (LAD) inversion results, the LAD joint inversion is found to be an accurate and robust method.

Introduction

When exploring near-surface structures, seismic and geoelectric methods are often applied. Inverting the recorded data sets separately may lead to incorrect parameter...
estimations for the underground structure, i.e. for an underground model with a high-velocity layer overlying a low-velocity layer, neither refraction nor reflection seismics alone is able to resolve the parameters of the low-velocity layer. Exploring the underground structure using geoelectric DC methods may also lead to problems in deriving the exact model, because of the ambiguity of solutions based on potential methods.

We propose avoiding these disadvantages by combining different geophysical data sets in one joint inversion. We confine ourselves to near-surface layers, i.e. structures having a maximum depth of around 20 metres. For the seismic exploration of these structures we use the dispersive surface-wave signal to determine the structural and petrophysical characteristics of the underground. (However, as is known from surface-wave and in-seam seismology, it should be noted that densities in general are irrelevant quantities.) Thus, the seismograms are analysed using the modified moving-window analysis (MMWA) after Kodera, de Villedary and Gendrin (1976). The data extracted from MMWA are inverted. The result of the inversion is the best-fitting dispersion curve. This curve contains all the important petrophysical and geometrical (structural) parameters of the near-surface underground. However, as well as an inversion of potential data, the inversion of seismic data sets may have inherent ambiguities and non-unique solutions (Pilant and Knopoff 1970; Kennett 1976).

In geoelectric exploration of near-surface structures, the DC method is frequently used. The geoelectric forward and inverse problems are well established for various arrays. Hence, the inversion of Schlumberger resistivity data, for example, has been discussed extensively both in the resistivity domain (Inman 1975; Petrick, Pelton and Ward 1977; Rijo et al. 1977) and in the kernel domain (Meinardus 1970; Hoverstein, Dey and Morrison 1982). Most of the difficulties resulting from equivalence and non-uniqueness problems in a geoelectric inversion are also well known today.

Stability and non-uniqueness problems can be sufficiently reduced by integrating physically different kinds of data into a single inversion procedure, or, in other words, by using joint inversion algorithms. In order to process d.c. resistivities and magnetotelluric (MT) data, Vozoff and Jupp (1975) introduced such a joint inversion method. They proved that some model parameters, which play an unimportant role in determining the solution of an individual (DC or MT) inversion problem, can lead to important parameters in a joint inversion, thus resulting in a well-resolved model.

Completely different physical data can also be integrated into a joint inversion if, at least, the measured data are influenced by a subset of the underground parameters. For example, when using seismic and geoelectrical data the common parameter in both methods is the depth. Lines, Schultz and Treitel (1987) used surface seismic data, sonic logs, VSP and gravity data to get stable joint inversion results. Zeyen and Pous (1993) published results of a magnetic/gravimetric joint inversion. In mine VSP and underground DC resistivity data were combined in an inversion
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procedure by Dobroka et al. (1991). The authors demonstrated that the accuracy and the reliability of the estimated model parameters (characterized by the model variances and correlations) are much better in a joint inversion than in an independent inversion.

In the joint inversion of VSP traveltimes and DC resistivities, the data set has a certain duality: the resistivity data, being dependent on all the petrophysical (geoelectric) and structural parameters, represent the complete geoelectric response of the (whole) geological structure, while seismic traveltimes of body waves, depending on a subset of seismic and structural parameters, contain information about seismic velocities and the location of boundary layers only along the raypaths between the source and the receivers. It is well known that the phase and group velocities of guided waves depend also on all of the (seismic) petrophysical and structural parameters of the geological model. So, by using dispersion data of guided waves, the duality in the two (geoelectric and seismic) data sets can be excluded.

In this paper both dispersive surface-wave data of the Rayleigh and Love type and DC resistivity data, recorded by Schlumberger, radial-dipole and two-electrode arrays, are involved in a joint inversion algorithm. The algorithm is tested using synthetic data. The numerical investigations are extended to both least-squares and robust inversion algorithms. A comparison is made between the various results.

Forward modelling

In order to analyse the features of the geoelectric/surface-wave joint inversion, we confine ourselves to horizontally layered (1D) geological models.

The solution of the geoelectric forward problem is well-known in this case for various electrode configurations (Koefoed (1979), Ghosh (1971) and Das and Ghosh (1974) for Schumberger and radial-dipole arrays; Das and Verma (1980) and Koefoed, Ghosh and Polman (1972) for two-electrode arrays). For an \( n \)-layered model, the apparent resistivities for different sounding measurements can generally be written as

\[
\rho_a = \rho_a(p_e, r),
\]

where the geoelectric layer parameter vector \( p_e \) is given by

\[
p_e = (h_1, \ldots, h_{n-1}, \rho_1, \ldots, \rho_n)^T,
\]

and \( h_i \) and \( \rho_i \) are, respectively, the thickness and resistivity of the \( i \)-th layer; \( r \) is the electrode distance defining the array spread.

Because the calculation of synthetic seismograms takes a lot of computer time, we shortened the process of obtaining data. Instead of taking the data from dispersion analysis of the seismograms, we calculate theoretical dispersion curves for Rayleigh and Love waves and then add noise to these curves. As a forward algorithm, the method of Schwab and Knopoff (1972) is used to derive the dispersion
curves for Rayleigh waves:

\[ S^{(R)}_g = S^{(R)}_g(\omega, \mathbf{p}_s), \]  

where

\[ \mathbf{p}_s = (h_1, \ldots, h_{n-1}, V_{p1}, \ldots, V_{pn}, V_{s1}, \ldots, V_{sn}, d_1, \ldots, d_n)^T \]

is the parameter vector of the seismic model parameters \( d, V_p \) and \( V_s \) (the mass density and the compressional- and shear-wave velocities, respectively), \( \omega \) is the frequency and \( S_g \) is the group-slowness. In order to calculate the group slownesses for Love waves,

\[ S^{(L)}_g = S^{(L)}_g(\omega, \mathbf{p}_s), \]  

a method developed by Buchanan (1987) is used.

In the linearized inversion method applied in this paper, the partial derivatives of the model response functions are needed. These quantities will be numerically determined, using the forward modelling formulae expressed by (1), (2) and (3).

The joint inversion algorithm

In order to formulate a joint inversion algorithm, it is necessary to combine the various data sets. Here, both the DC resistivities (Schlumberger, radial-dipole and two-electrode arrays) and the Rayleigh and Love wave dispersion characteristics represent data sets of a completely different physical nature. The material characteristics, determining the geoelectric response of the ‘earth’ model, are different from those determining the seismic response. In order to be sure that geoelectric and seismic data can be combined in a joint inversion scheme, it is assumed that the layer-interfaces are the same for both the geoelectric and seismic petrophysical parameters.

In order to derive the combined response function, we introduce the parameter vector \( \mathbf{X} \) as a combination of \( \mathbf{P}_s \) and \( \mathbf{P}_e \) in the joint problem as

\[ \mathbf{X} = (h_1, \ldots, h_{n-1}, V_{p1}, \ldots, V_{pn}, V_{s1}, \ldots, V_{sn}, d_1, \ldots, d_n, \rho_1, \ldots, \rho_n)^T, \]  

and write the responses in the form

\[ Y^{\text{calc}}_i = Y(\mathbf{X}, s_i), \]  

where \( s_i \) are the electrode separations for geoelectric arrays and frequencies for surface-wave slownesses. Using the notation \( Y^{\text{obs}}_i \) for the observed data, we can write the combined response function and data vector of the joint problem in a
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more detailed form as

\[
Y(X, s_i), \quad Y_i^{\text{obs}} \Rightarrow \begin{cases}
\rho_{a_i}^{(s)}(p_e, r_i), & i = 1, \ldots, N1 \\
\rho_{a_i}^{(D)}(p_e, r_i), & i = N1 + 1, \ldots, n2 \\
\rho_{a_i}^{(T)}(p_e, r_i), & i = n2 + 1, \ldots, n3 \\
s_{R}^{(R)}(p_s, \omega_i), & i = n3 + 1, \ldots, n4 \\
s_{R}^{(L)}(p_s, \omega_i), & i = n4 + 1, \ldots, N
\end{cases}
\]  

(6)

where \( n_2 = N_1 + N_2, \ n_3 = n_2 + N_3, \ n_4 = n_3 + N_4, \ N = n_4 + N_5 \) and \( N_1, N_2, N_3, N_4, N_5 \) denote the total number of data in the individual data sets and \( N = n_5 \) the total number of data.

The model responses given in (5) are non-linear functions of the elements of the parameter vector (4). In order to solve the inverse problem, iterative methods are frequently applied, and the current model is improved successively, until an appropriate stop criterion is met. Let \( X_0 \) be an initial estimate and let the responses (6) be approximated at \( X = X_0 + \delta X \) by the first-order Taylor expansion as

\[
Y_i^{\text{calc}} = Y_i^{(0)} + \sum_{j=1}^{M} \left( \frac{\partial Y_i^{\text{calc}}}{\partial X_j} \right)_{X=X_0} \delta X_j,
\]

where \( Y_i^{(0)} = Y(X_0, s_i) \) and \( M \) is the number of the model parameters. The solution of the inverse problem is usually connected to the minimization of a certain norm of the error vector:

\[
e = Y^{\text{obs}} - Y^{\text{calc}}.
\]

In a joint inversion procedure, the elements of the vector \( e \) can have different physical dimensions and various orders of magnitude. Thus, to have stable results, normalization is needed. Following Jupp and Vozoff (1975) and Dobroka et al. (1991), we normalize the error vector elements as \( f_i = e_i / Y_i^{(0)} \) and introduce the relative prediction error

\[
f = y - G x,
\]

where

\[
y_i = \frac{Y_i^{\text{obs}} - Y_i^{(0)}}{Y_i^{(0)}}, \quad x_j = \frac{\delta X_j}{X_j^{(0)}}
\]

and

\[
G_{ij} = \frac{X_i^{(0)}}{Y_i^{(0)}} \left( \frac{\partial Y_i^{\text{calc}}}{\partial X_j} \right)_{X=X_0}
\]

The minimization of the \( L_p \)-norm

\[
E \left( \sum_{i=1}^{N} |y_i - \sum_{j=1}^{M} G_{ij} x_j|^p \right)^{1/p}
\]

(10)
of the vector $f$ leads to the normal equation

$$G^T R G x = G^T R y,$$

with the diagonal matrix

$$R_{ij} = \begin{cases} \frac{1}{f_i^{p-2}}, & \text{if } j = i, \\ 0, & \text{otherwise}, \end{cases}$$

(Scales et al. 1988). In the case of $p = 2$, (11) gives the well-known (linear) normal equations

$$G^T G x = G^T y,$$

valid for least-squares (LSQ), while for $p \neq 2$, (11) is non-linear in $x$. In the $k$th step of an iterative procedure, the elements of the matrix $R$ can be replaced approximately by those calculated in the $(k-1)$th step. Thus (11) gives the set of linear equations

$$G^T R^{(k-1)} G x^{(k)} = G^T R^{(k-1)} y,$$

corresponding to the norm equations of a weighted least-squares algorithm with the weight matrix $W = R^{(k-1)}$. Equation (14) defines the so-called iteratively reweighted least-squares (IRLS) algorithm applicable for any case with $p \neq 2$. In the inversion praxis, the case of $p = 1$ the least absolute deviation (LAD) is often used as a robust inversion algorithm. From (12), it follows that the contribution of outlier data to the solution is weighted towards zero in the LAD inversion.

**Numerical investigations**

In the numerical experiments we confine ourselves to a four-layered model with parameters as given in Table 1a. Apparent resistivities for Schlumberger, radial-dipole and two-electrode arrays were calculated for 24 logarithmically equidistant electrodes (distances from 1.6 m to 320 m). The Rayleigh and Love surface-wave group slownesses were calculated for frequencies between 3 Hz and 80 Hz in 1 Hz steps.

In order to simulate real data (Fig. 1), three geoelectric data sets were generated (Figs 1a and b). In Figs 1a and c, 2% random noise of Gaussian distribution was added to the theoretical geoelectric and seismic data, respectively. In Figs 1b and d, an additional error of not more than 40% was added to a randomly selected 20% portion of the geoelectric and seismic data. These sets include the outliers.

Furthermore, an additional seismic data set was generated using synthetic seismograms (Fig. 2), calculated on the basis of the model data given in Table 1a. From these seismograms, group slownesses were extracted using the method of modified moving-window analysis (Fig. 3, marked by the symbol $\times$). Thus the data sets for the inversion are: (i) geoelectric and seismic data with 2% noise; (ii) geo-
Table 1. (a) The petrophysical and geometrical layer parameters of the exact four-layer model. The half-space represents the 4th ‘layer’.
(b) The petrophysical and geometrical layer parameter of the initial four-layer model. The half-space represents the 4th ‘layer’.

(a)

<table>
<thead>
<tr>
<th>H (m)</th>
<th>P-Velocity (m/s)</th>
<th>S-Velocity (m/s)</th>
<th>Density (g/cm³)</th>
<th>Resistivity (Ωm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>1500</td>
<td>700</td>
<td>1.7</td>
<td>20</td>
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<td>2500</td>
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(b)

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<th>H (m)</th>
<th>P-Velocity (m/s)</th>
<th>S-Velocity (m/s)</th>
<th>Density (g/cm³)</th>
<th>Resistivity (Ωm)</th>
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<td>8.0</td>
<td>2000</td>
<td>1000</td>
<td>2.1</td>
<td>40</td>
</tr>
<tr>
<td>half-space</td>
<td>2700</td>
<td>1200</td>
<td>2.3</td>
<td>120</td>
</tr>
</tbody>
</table>

electric and seismic data with noise and outliers; and (iii) geoelectric with noise in combination with the Rayleigh group slowness data depicted in Fig. (3).

To qualify the solution of the inverse problem, we used the relative distance

\[ D = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( \frac{Y_{i}^{\text{obs}} - Y_{i}^{\text{calc}}}{Y_{i}^{\text{calc}}} \right)^2} \]  

(15)

between the observations and the data calculated for the estimated model. In addition, we used the relative distance

\[ d = \sqrt{\frac{1}{M} \sum_{j=1}^{M} \left( \frac{X_{j}^{\text{exact}} - X_{j}^{\text{estimated}}}{X_{j}^{\text{exact}}} \right)^2} \]  

(16)

between both the exact and the estimated model and the elements of the model covariance matrix, defined as

\[ \text{cov} = \sigma^2 (Q^T G)^{-1}, \]  

(17)

for least-squares inversion (Menke 1984).

The diagonal elements of the matrix (17) were used to determine the (percentage) estimation error of the model parameters. The correlation matrix, defined as

\[ \text{corr}_{ij} = \frac{\text{cov}_{ij}}{\sqrt{\text{cov}_{ii} \text{cov}_{jj}}}, \]  

(18)
Figure 1. Synthetic data sets. (a) Apparent resistivity data for Schlumberger (○), radial-dipole (▲) and two-electrode (×) arrays. The data set contains 2% Gaussian noise. (b) Apparent resistivity data for Schlumberger (○), radial-dipole (▲) and two-electrode (×) arrays. The data set contains 2% Gaussian noise and an additional error of not more than 40% added to a random selected 20% portion of the data. (c) Slowness data for Rayleigh (○) and Love (×) waves. The data set contains 2% Gaussian noise. (d) Slowness data for Rayleigh (○) and Love (×) waves. The data set contains 2% Gaussian noise and an additional error of not more than 40% added to a randomly selected 20% portion of the data.

and its mean spread,

\[ S = \sqrt{\frac{1}{M(M-1)} \sum_{j=1}^{M} \sum_{i=1}^{M} (\text{corr}_{ij} - \delta_{ij})^2}, \]  

(19)

were used to characterize the correlation between the estimated model parameters (\(\delta_{ij}\) being the Kronecker symbol).
Figure 2. FD seismograms calculated for the model (used for numerical investigations, Table 1a) at various distances, starting 10 m from the source. Only the vertical displacements are plotted.

**Least-squares joint inversion results**

In order to demonstrate the possible advantages of a joint inversion we analyse the sensitivities

\[
\phi_j = \frac{\text{parameter}_j \times \frac{\partial (\text{data curve})}{\partial (\text{parameter}_j)}}{\text{data curve}}
\]

introduced by Gyulai (1990). The sensitivities of the parameters \(\rho_1\) and \(\rho_3\) for the different sounding arrays are shown in Fig. 4. Using a joint inversion of the data
from the Schlumberger, the radial-dipole and the two-electrode soundings, the reliability of the inversion result increases, the more the sensitivity curves of \( \rho_1 \) and \( \rho_3 \) for the different sounding methods differ from each other. In our case, the sensitivities differ appreciably for at least two arrays at all electrode spacings. Thus, we conclude that it is worth trying to apply a geoelectric/geoelectric joint inversion.

*Geoelectric joint inversion results.* The individual inversion results of any of the Schlumberger, radial-dipole and two-electrode data sets (containing only 2% Gaussian noise, defined as (i) in Section 4) were unstable if all the model parameters were variable. With \( \rho_3 \) as a fixed parameter, only the inversion of the Schlumberger data gave acceptable results. In this numerical experiment (as in all those following) the model data shown in Table 1b were used as an initial model. In Fig. 5, the relative model distances (defined by (15)) at various steps during the iter-
A joint inversion algorithm

Figure 4. Sensitivities of the Schlumberger (dotted line), the radial-dipole (broken line) and the two-electrode (solid line) apparent resistivities at various electrode spacings. The sensitivity curves are calculated with respect to the $\rho_1$ (Fig. 1a) and $\rho_3$ (Fig. 1b) resistivities, respectively.

The single inversion of the Schlumberger resistivity is represented by the curve (□). For the first 39 steps of iteration the model distance decreases but after step 39 a strong increase to 29% model distance can be observed. Thus, it is obvious that the inversion result for the single inversion is not reliable.

This result can be improved by combining Schlumberger and radial-dipole resistivities in the geoelectric/geoelectric joint inversion algorithm. In Fig. 5, the curve (△) shows the course of this joint inversion process. As in the case of the single inversion, this curve also has a minimum. However, it approaches a stable plateau earlier than the Schlumberger curve and the model distance is much smaller (13%). Involving the third geoelectric data set (in this case, two-electrode apparent resistivities), the parameter estimation is further improved. This is
Figure 5. Mean relative distances between the exact and the estimated models at various iteration steps. The curves marked by □, Δ and * refer respectively to: the independent inversion of Schlumberger data; the joint inversion of Schlumberger and radial-dipole data; and the joint inversion of Schlumberger, radial-dipole and two-electrode apparent resistivities. At the zeroth iteration, the relative distance between the exact and the initial model is shown. The \( \rho_3 \) resistivity was considered as a constant parameter during the (LSQ) inversion.

demonstrated by the curve (*) in Fig. 5. At the end of this inversion process the distance between the exact and the estimated models is only 5%. This represents a decrease of a factor of about 5 relative to the independent (single) Schlumberger data inversion.

It is well known that apparent resistivities measured in various arrays can be transformed into each other with sufficient accuracy. Thus, it is possible to use only a single inversion procedure, developed for a particular, e.g. Schlumberger, resistivity data set and to apply a transformation to other types of data sets. However, based on the results of Fig. 5 and as an inversion strategy, we suggest using (when dealing with various kinds of resistivity data) the geoelectric joint inversion. A geoelectric joint inversion leads to more reliable results and should be preferred to subsequent individual inversions of the various data sets. It should be noted that this recommendation holds for inversions carried out only in the apparent resistivity domain.

In Table 2 both the estimated parameters of the joint inversion including their percentage errors (upper part) and the elements of the correlation matrix (lower
Table 2. The estimated model parameters with the (percentage) estimation errors and the model correlation matrix in the separate geoelectric joint inversion.

<table>
<thead>
<tr>
<th>H (m)</th>
<th>Resistivity (Ωm)</th>
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<tbody>
<tr>
<td>5.1 (29%)</td>
<td>20.0 (1%)</td>
</tr>
<tr>
<td>3.7 (127%)</td>
<td>9.2 (98%)</td>
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<tr>
<td>9.1 (37%)</td>
<td>50.0 (fix)</td>
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<td>100.3 (1%)</td>
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<table>
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<th>H1</th>
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<tr>
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<td>-0.9</td>
<td>1.0</td>
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<td>-0.5</td>
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<tr>
<td>-1.0</td>
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<td>-0.2</td>
<td>1.0</td>
</tr>
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part) are shown. The ‘high’ values of the off-diagonal elements of the correlation matrix near ±1 indicate that the estimated model is not resolved to a high degree (the mean spread has a relatively high value of 0.643), although the separated geoelectric joint inversion is used.

Seismic joint inversion results. The individual inversion of Rayleigh or Love wave dispersion data, especially in multilayered structures, requires an a priori knowledge of the density relation. Since small uncertainties in the layer densities have little effect on the dispersion characteristics, the a priori (even rough) density relation can usually be considered as accurate enough to give an acceptable estimate during the inversion procedure. Thus, in the inversion of Rayleigh and Love wave slowness data, the mass densities were assumed to be given (known) parameters. These ‘known’ densities given in Table 1a are representative values for near-surface layers.

The result of the joint inversion (Rayleigh and Love waves) of the model slowness data containing 2% Gaussian noise, is shown in Fig. 6. The model distance (curve ▲) rapidly decreases during the first three iterations. After the third iteration, the model distance slowly increases; after 33 iterations, it approaches a stable ‘minimum level’, which amounts to 12.3%. Figure 6 also depicts both the data distances (curve ○) and the norm of the parameter correction vector (curve *) at each iteration step. The two curves tend to zero; this indicates a stable inversion.

The estimated model parameters and their errors are given in Table 3a. The mean estimation error is 8.9% and the mean spread of the correlation matrix is 0.505. The relatively high values of the mean estimation error and of the mean
Figure 6. Mean relative model distances (curve ▲), data distances (curve ◀) and the norm of the parameter correction vector (curve ▼) at various iteration steps in the (LSQ) joint inversion of Rayleigh and Love wave slowness data. The model distance in the zeroth iteration refers to the initial model (Table 1b).

Table 3. The estimated model parameters with the (percentage) estimation errors: (a) in a separate seismic joint inversion and (b) in a geoelectric/surface-wave seismic joint inversion.

(a)  

<table>
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<th>H (m)</th>
<th>P-Velocity (m/s)</th>
<th>S-velocity (m/s)</th>
<th>Density (g/cm³)</th>
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<td>1294 (9%)</td>
<td>694 (2%)</td>
<td>1.7 (fix)</td>
</tr>
<tr>
<td>3.9 (15%)</td>
<td>2070 (10%)</td>
<td>812 (5%)</td>
<td>1.8 (fix)</td>
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<td>8.8 (9%)</td>
<td>1959 (7%)</td>
<td>1141 (2%)</td>
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<td>2484 (15%)</td>
<td>1319 (1%)</td>
<td>2.3 (fix)</td>
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</table>

(b)  

<table>
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<th>P-Velocity (m/s)</th>
<th>S-Velocity (m/s)</th>
<th>Density (g/cm³)</th>
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<td>4.9 (3%)</td>
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<td>700 (1%)</td>
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<tr>
<td>4.0 (7%)</td>
<td>2045 (10%)</td>
<td>906 (2%)</td>
<td>1.8 (fix)</td>
<td>9.8 (6%)</td>
</tr>
<tr>
<td>9.6 (8%)</td>
<td>2000 (7%)</td>
<td>1181 (1%)</td>
<td>2.1 (fix)</td>
<td>50.0 (fix)</td>
</tr>
<tr>
<td>half-space</td>
<td>2439 (16%)</td>
<td>1320 (1%)</td>
<td>2.3 (fix)</td>
<td>100.5 (1%)</td>
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</tbody>
</table>
spread indicate that it is worth trying to improve the parameter resolution by a joint inversion of geoelectric and seismic data.

*Geoelectric/surface-wave joint inversion results.* In order to test the geoelectric/surface-wave joint inversion procedure, the data sets containing 2\% Gaussian noise (Figs 1a and c) were used. The densities and the \( \rho_3 \) resistivity were again kept constant. The model distance, the data distance and the norm of the parameter correction vector at each iteration step is shown in Fig. 7. It can be seen that in the geoelectric/seismic joint inversion, the parameter estimation is better compared to the results of the seismic/seismic joint inversion. The distance between the estimated and the exact models is under 10\%. The estimated parameters and estimation errors are given in Table 3b, while the correlation matrix is shown in Table 4. Compared to Table 2, the values of the off-diagonal elements of the correlation matrix near \( \pm 1 \) are appreciably reduced. This indicates that the model is much better resolved than in the case of a separate geoelectric or surface-wave joint inversion. The geoelectric and seismic model distances are 0.020 and 0.081, respectively. The mean estimation error is 4.8\% and the mean spread is 0.345. Relative to

![Figure 7](image.png)
Table 4. The model correlation matrix in a geoelectric/surface-wave seismic (LSQ) joint inversion. The densities are the resistivity \( \rho_3 \) (Table 1b) are kept constant during the iterations.

<table>
<thead>
<tr>
<th></th>
<th>H1</th>
<th>H2</th>
<th>H3</th>
<th>VP1</th>
<th>VP2</th>
<th>VP3</th>
<th>VP4</th>
<th>VS1</th>
<th>VS2</th>
<th>VS3</th>
<th>VS4</th>
<th>R01</th>
<th>R02</th>
<th>R04</th>
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</thead>
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<td>0.5</td>
<td>0.5</td>
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<td>0.1</td>
<td>-0.6</td>
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<td>-0.3</td>
<td>-0.1</td>
<td>-0.3</td>
<td>-0.1</td>
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<tr>
<td>H2</td>
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<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>0.3</td>
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<tr>
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<td>0.4</td>
<td>0.3</td>
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<tr>
<td>VP1</td>
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<td>0.4</td>
<td>0.2</td>
<td>1.0</td>
<td>0.6</td>
<td>0.2</td>
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<td>0.3</td>
</tr>
<tr>
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<td>0.4</td>
<td>1.0</td>
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<tr>
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<td>0.1</td>
<td>0.0</td>
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<tr>
<td>VS4</td>
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<td>0.2</td>
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<td>1.0</td>
<td>0.1</td>
<td>0.0</td>
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<tr>
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<td>1.0</td>
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<td>0.4</td>
<td>0.3</td>
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<td>0.3</td>
<td>1.0</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
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<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
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<td>R04</td>
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<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The model correlation matrix in a geoelectric/surface-wave seismic (LSQ) joint inversion. The densities are the resistivity \( \rho_3 \) (Table 1b) are kept constant during the iterations.
the separate (independent from seismic) geoelectric joint inversion, the model distance is appreciably reduced in the combined (seismic/geoelectric) joint inversion. The same holds for the separate (independent from geoelectric) seismic joint inversion. Both the mean estimation error and the mean spread are also reduced in the seismic/geoelectric joint inversion.

Because of the stability of the algorithm, the $\rho_3$ resistivity need not necessarily be constant. The results of the joint inversion in such a test are shown in Fig. 8. For comparison, the curve (□) and the curve (△) give the model distance in a separate geoelectric and a separate seismic joint inversion, respectively. The curve (⁎) refers to the combined seismic/geoelectric joint inversion. The seismic model distance is 0.081, the geoelectric model distance is 0.019, the mean estimation error is 7.4% and the mean spread is 0.362. It can be seen that allowing $\rho_3$ to be a free parameter results in a sufficient increase of the model distance in the separate geoelectric inversion. The geoelectric model distance in the combined joint inversion is nearly the same as with constant $\rho_3$.

**Joint LSQ inversion of data with non-Gaussian noise.** It is known that a least-squares inversion is very sensitive to sparsely distributed large errors (outliers) in the data set. In order to investigate the problem in the case of a seismic/geoelectric

![Figure 8](image-url)

**Figure 8.** Mean relative distances between the exact and the estimated models at various iteration steps in a separate geoelectric (curve □), separate seismic (curve △) and geoelectric/surface-wave seismic (LSQ) joint inversion (curve ⁎). The model distance in the zeroth iteration refers to the initial model (Table 1b).
joint (LSQ) inversion we used the geoelectric and seismic data sets (Figs 1b and d) generated with noise and outliers. The model distances, the data distances and the norm of the parameter correction vector are plotted at each step of iteration in Fig. 9. Stable inversion results were found, but compared to curve (*) in Fig. 8, the model distance is much higher. Thus, to handle outlier data, it is necessary to use robust inversion methods in joint inversion algorithms.

Robust joint inversion results

The LAD method, which minimizes the $L_1$-norm of the error vector, is often used as a robust method in the inversion of geophysical data. In our numerical investigations, we applied the IRLS technique defined in (14) for LAD seismic/geoelectric joint inversion of the data set with noise and outliers (Figs 1b and d). The result is shown in Fig. 10. It can be seen that the LAD method results in a sufficient improvement with respect to the parameter estimation, if compared to the LSQ inversion. In the case of input data with Gaussian noise, the LAD inversion shows similar results to the LSQ method. Thus, an $L_1$-inversion using an IRLS scheme can also be considered as a powerful and robust method in a geoelectric/surface-

![Figure 9. Mean relative model distances (curve ▲), data distances (curve ○) and the norm of the parameter correction vector (curve *) at various iteration steps in the geoelectric/surface-wave seismic (LSQ) joint inversion of data sets containing non-Gaussian noise (outliers). The model distance in the zeroth iteration refers to the initial model (Table 1b).](image-url)
A joint inversion algorithm

Figure 10. Mean relative model distances (curve ▲) and the norm of the parameter correction vector (curve ▼) at various iteration steps in the geoelectric/surface-wave seismic (LAD) joint inversion of data sets containing non-Gaussian noise (outliers). The model distance in the zeroth iteration refers to the initial model (Table 1b).

wave joint inversion. This technique requires similar programming efforts and computation time as a weighted LSQ inversion.

Seismic/geoelectric joint inversion using ‘moving-window data’ derived from synthetic seismograms. In the above tests both the geoelectric and seismic dispersion data were model data, calculated for the geological model depicted in Table 1a. In the case of real recordings, group slownesses are not directly available; they have to be determined from seismograms. So, for testing the seismic/geoelectric joint inversion algorithm, it is also necessary to use seismic slowness data derived from seismograms. The seismic traces, computed by means of an FD method for our geological model, are shown in Fig. 2. In order to extract group slowness data we applied the method of Kodera et al. (1976). The result of the moving-window analysis is drawn in Fig. 3. For all the frequencies shown, the slowness belonging to the maximum amplitude was selected as input data for the joint inversion (crosses in Fig. 3). The geoelectric data set again contains 2% noise.

There is no reason to assume that the errors in moving-window data follow a Gaussian distribution. Thus, in the combined geoelectric/seismic inversion we used only the LAD method. To find an acceptable parameter estimation for the
above data set, the velocity $V_{p4}$ of the underlying half-space should be kept constant during the inversion. Usually $V_{p4}$ represents the solid rock and it can easily be determined from refraction and reflection seismics. The result is shown in Fig. 11. It can be seen that the robust geoelectric/surface-wave joint inversion algorithm also leads to stable and reliable results for dispersion data derived from seismograms.

Conclusions

A joint inversion scheme for the interpretation of geoelectric and surface-wave seismic data was introduced and analysed.

It was shown that the inversion of various kinds of apparent resistivity data, derived from Schlumberger, radial-dipole and two-electrode arrays, by a geoelectric joint inversion procedure is worth applying in the process of a joint geoelectric/seismic inversion.

Compared to the separate geoelectric or seismic joint inversion, the combination of the resistivity and surface-wave slowness data in a joint inversion algorithm

![Figure 11](image_url)

**Figure 11.** Mean relative model distances (curve ▲) and the norm of the parameter correction vector (curve *) at various iteration steps in the geoelectric/surface-wave seismic (LAD) joint inversion of data sets containing moving-window slowness data. The model distance in the zeroth iteration refers to the initial model. The resistivity $\rho_3$ and also the velocity $V_{p4}$ are kept constant during the inversion.
results in a much better parameter estimation: the geoelectric and seismic model distances, as well as the mean estimation errors, are appreciably reduced.

Using an IRLS (iteratively reweighted least-squares) algorithm, it was shown that the LAD (least absolute deviation) method can be considered as a robust and powerful algorithm for a geoelectric/surface-wave joint inversion. It also leads to acceptable results in the case of slowness data determined from seismograms.

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