INVESTIGATING THE PROBLEM OF EQUIVALENCE IN KINEMATIC REFRACTION INVERSION

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INTRODUCTION

Equivalence problem is common in seismic refraction interpretation as in field propagation time data of the layer-intercepting waves are measured therefore we only get the quotients of the layer thickness and the velocity for each layer. This way from the same measured data many different geological models can be calculated. This problem arises especially in the interpretation of multilayered cases where the parameters vary laterally. Such an inversion method was developed in the Department of Geophysics, University of Miskolc for multilayered structures, where lateral changes in the physical parameters (e.g. layer thicknesses, propagation velocities, etc.) are described by adequately chosen continuous basic functions expanded in series. The function coefficients are estimated with qualified linearized LSQ and other types of inversion [1] [2] [3].

Our inversion method was investigated earlier to prove that it is capable of interpreting geologic structures with laterally slowly changing layer thicknesses [4]. In the recent paper the handling of the equivalence problem of refraction methods is investigated. This is an important question as in a laterally inhomogenous medium the results of the interpretation can be hardly reliable. The same best fitting of the measured and calculated data can result in significantly different models.

The equivalence problem of seismic refraction methods is usually reduced by using joint or constrained inversion methods [5] [6] [7]. The main point in these approaches is that they use data measured with some other geophysical methods (e.g. geoelectric, surface waves, etc.). This way the number and types of data are highly increased, giving us a more definite result.

In this paper the problem of equivalence in refraction seismic acquisition is attempted to be solved without any other geophysical method. It is investigated how much our inversion method is sensitive for the usual equivalence problem of the refraction method and what are the ways of its handling.

THE INVERSION METHOD

The firstly used inversion method of refraction time data that used function description was developed by Bernabini et al. [8]. They have calculated the coefficients of fourth degree power functions for describing the refractors of a multi-layered geological model, with laterally unvarying velocities.

As we know the geological structures in nature, it is obvious that an inversion method capable of dealing with slow lateral changes both in thicknesses and velocities, has to be developed [2]. With the help of this method the determination of refracting horizons and layer velocities are possible. Lateral changes in both wave propagation velocities in the media and layer thicknesses are described by series expansions using adequately chosen basic functions. The $p_i(x)$ physical parameters of the investigated model (e.g. thicknesses and velocities) can be defined as follows (Eq. 1).

$$p_i(x) = \sum_{j=1}^{Ji} C_{ij} \cdot F_{ij}(x) \qquad (1)$$

In the equation i represents the layer numbers, J_i the number of functions defining the i^{th} parameter, $F_{ij}(x)$ the j^{th} basic function of the i^{th} parameter and C_{ij} the j^{th} expansion coefficient of the i^{th} parameter. The basic function $F_{ij}(x)$ in case of trigonometric series takes the form of $sin(j*2\pi x_i/X)$ and $cos(j*2\pi x_i/X)$ (X means the length of the profile here). C_{ij}-s do not depend on the distance x along the profile and therefore they are suitable for determination (estimation) by inversion.
In solving the direct problem some assumptions and approximations have to be taken. Firstly, high frequency limit is assumed, this way the parameters of the model might vary only slowly compared to the wavelength, so the radius of boundaries' curvature has to be significantly higher than the horizon depth. Secondly, only those rays are considered in the calculations that propagate along the layer boundaries, therefore no penetration (diving) effect exists. Finally, ray traces within each layer are approximated with straight lines, even in layers where the velocity is changing. But with these approximations the running time of the method turns to be very short (e. g. only a few minutes for 100 iterations with 1500 observed data and 40 unknowns).
The estimation of the C_{ij} coefficients has to be carried out in solving the inverse problem. After that it is necessary to calculate the physical parameters of the 2-D model from the estimated C_{ij} coefficients along the profile. The function coefficients are calculated by a qualified linearized LSQ inversion, according to which the nonlinear problem can only be solved iteratively. The above described inverse problem is formulated in the following well-known form (Eq. 2):

$$\vec{c} = \left(\underline{\underline{G}}^T \underline{\underline{G}} + \lambda \underline{\underline{I}}\right)^{-1} * \underline{\underline{G}}^T \vec{t} \qquad (2)$$

Here $\vec{c}$ means the correction vector of the coefficients (compared to the previous estimation), $\vec{t}$ is the vector of the differences between the observed and (from the estimated coefficients) calculated first arrival times, and $\underline{\underline{G}}$ is the (Jacobi-) matrix of the partial derivatives of the traveltimes according to the C_{ij} function coefficients. $\underline{\underline{I}}$ represents the unit matrix, and λ is the scalar damping factor.
The investigations are qualified with the following standards: the relative deviation D in the data space (Eq. 3) and the relative deviation d in the model space (Eq. 4).

$$D = \sqrt{\frac{1}{I}\sum_{i=1}^{I}\left(\frac{T_i^{(observed)} - T_i^{(calculated)}}{T_i^{(calculated)}}\right)^2} \qquad (3)$$

$$d = \sqrt{\frac{1}{M}\frac{1}{(2N-1)}\sum_{m=1}^{M}\sum_{n=1}^{2N-1}\left(\frac{p_n(x_m)^{(exact)} - p_n(x_m)^{(estimated)}}{p_n(x_m)^{(exact)}}\right)^2} \quad (4)$$

(Here i denotes the number of time data, m denotes the number of source points, x_m the distance of the m^{th} source point along the profile, n the number of model parameters).

INVESTIGATIONS ON A SYNTHETIC GEOLOGICAL STRUCTURE

For the investigation of the equivalence problem, a simple 3-layered synthetic geological model was created with laterally changing velocity of the second layer. Only one parameter varies laterally because it is easier to investigate the equivalence problem and all the effects of one changing parameter at a time. This geological section is 250 metres long in x direction along which 26 sources are placed in every 10 metres and 126 geophones in every 2 metres. This way the whole section is properly covered by rays, no problem could occur because of the lack of data. The propagation velocities in the layers are the following: 400 m/s in the first layer, 1000-2000 m/s varying velocity in the second layer and 2400 m/s in the third one. The investigated synthetic model can be seen in Figure 1. The values of velocities are described by different colours explained in the colour scale.

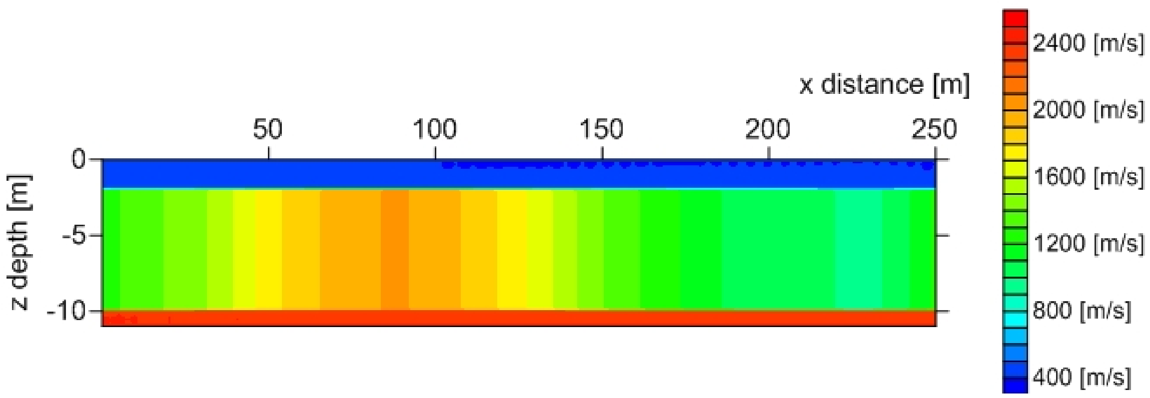

Figure 1
The initial synthetic multilayered model for our equivalence problem investigation. The velocity of the second layer is laterally changing.

The investigation was carried out using synthetic data computed on the model in *Figure 1*, with FD-Vidale type ray tracing, a method based on FD approximation of the eikonal equation for calculating synthetic traveltimes using the ReflexW software by Sandmeier [9] [10]. The calculated synthetic data (1566 data) were used in the inversion method as quasi "measured data" without additional error. An approximately 1% error still presents that arises from the difference between the two used forward problem solution methods during the inversion process. As a

target model the initial synthetic model was set and this way the difference or similarity between the initial and calculated structure became easily definable.
It is investigated how close would the estimated models lie to the initial model (*Figure 1*).

RESULTS

For the investigation the same synthetic model was used but in the interpretation of these quasi measured data the used coefficients in the inversion process were different. During the inversion process all parameters of the layers were allowed to vary laterally. Many different combinations of coefficients were applied and in most cases the equivalence problem was just eye-catching.
There were no problem in the estimation of the first layer in any case; both its velocity and thickness have just the same values as of the initial model regardless of number of used coefficients in the series expansion, or the types of the used functions in the inversion method. The situation was almost the same in case of the third layer. The only difference is that the velocity of the third layer is allowed to vary only really slowly – compared to the other parameters of the section – but slowly changing velocity in the base layer is a common feature in nature as well.
In case of the second layer, strong equivalence problem occurred for its laterally changing velocity and thickness. If we tried to describe these 2 parameters with the same number of coefficients (13) in a series, bad result were obtained (*Figure 2*). Although relative deviation in data space was only D=0.9 %, its misfit is just eye-catching. The relative deviation in model space was d=11.7%.

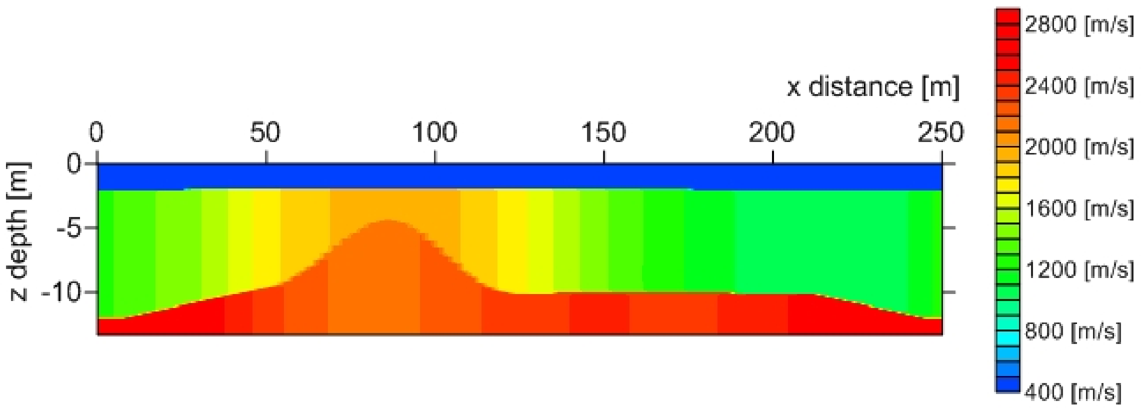

Figure 2
The equivalence problem of the thickness and velocity parameters in the 2^{nd} layer of the investigated structure. The flat layer turned to be an anticline and the velocities of the lower layers have also changed compared to the initial model (Figure 1).

We have found out that it is the number of coefficients of the thickness and the velocity of the second layer (these are in equivalence) that have to differ significantly. In our case if you choose a coefficient for the layer thickness that can describe the laterally changing velocity (or even bigger) then high equivalence appears. In an example of equivalence (*Figure 2*) both the thickness and velocity of the second layer were interpreted by Fourier series with 13 coefficients. This explains the anticline form of the resulted second layer boundary instead of a flat one. (The numbers of coefficients for this interpretation were 5, 13 for the layer thicknesses and 5,13,5 for the velocities; 100 iteration steps were done.)

In reducing equivalence problem besides choosing significantly different coefficients for the problematic parameters another option is to describe them with different types of functions. The investigated inversion method makes it possible to use power function or Fourier series. In *Figure 3* the reduced equivalence problem can be seen through two examples. In the first one Fourier series were used for all the parameters, the numbers of coefficients were 5 for layer thicknesses and 5, 13 for the upper velocities. The relative deviations were D=0.6%; d=1.4% (Eq. 3); its results can be seen in Figure 3a. In the second interpretation power function was used for the description of the layer thicknesses (nr. of coefficients: 5) and Fourier series for describing the velocities of the upper layers (nr. of coefficients: 5, 13). The relative deviations were D=0.5 %; d=3.3 %; its results are shown in Figure 3b. In both cases 100 iteration steps were done and for the velocity of the third layer power function was used with 2 coefficients for allowing a really slowly changing base velocity.

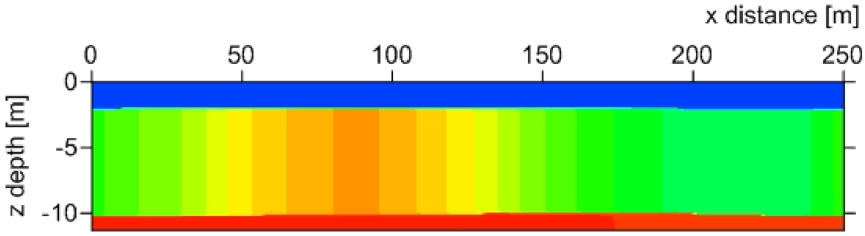

Figure 3a

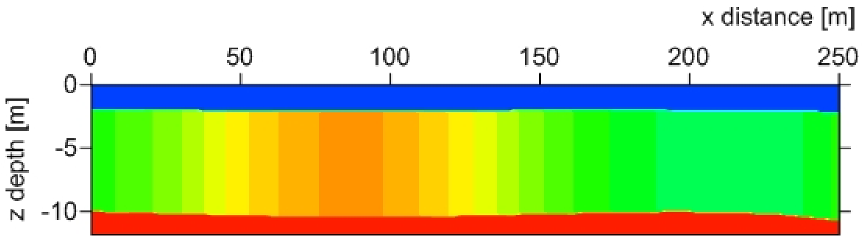

Figure 3b

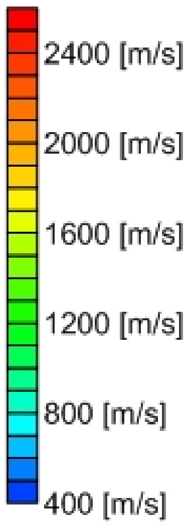

Figure 3

Successfully reduced equivalence problem in the investigated geological structure. ***Fig. 3a*** *shows the usage of different number of coefficients and* ***Fig. 3b*** *shows the usage of different function types for the interpretation of parameters.*

CONCLUSIONS

The problem of equivalence is a considerable problem in the interpretation of seismic refraction data; it is usually solved by using joint inversion. With the usage of our inversion method that describes the laterally varying geological parameters (layer thickness and velocity) with Fourier series or power functions, the problem of equivalence can be reduced without other types of measured data (joint inversion). As a result of our investigation the key for the problem was the usage of different function types or just different number of coefficients of the same functions in the inversion process for the parameters in equivalence. This way this simple, quick inversion method proved that without any other geophysical measurements it is possible to interpret a relatively complex geological structure well and reliably.

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