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Investigations on Kinematic Refraction Inversion at Different Geological Models

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SUMMARY

In this paper a refraction inversion technique developed in the Department of Geophysics, University of Miskolc is investigated for different laterally changing geological models. It is important because the solution of the forward problem uses several approximations for significant reduction of the calculation time and these can affect the inversion results. The effects of these approximations depend on the investigated geological structure. According to our investigations it can be stated that the applied approximation gives very good results in case of slowly changing structures, while even in case of a highly changing model the inversion technique gives a result where the target model is recognizable.

Introduction

In the Department of Geophysics, University of Miskolc several inversion methods were developed for multilayered cases based on the fact that lateral changes in the physical parameters (e.g. layer thicknesses, propagation velocities and resistivities) are described by adequately chosen continuous basic functions expanded in series. Coefficients of the functions are estimated with a qualified linearized LSQ and other types of inversion. This approximation was successfully applied in inversion and joint inversion of geoelectric and seismic data (Dobróka, 1997) (Gyulai and Ormos, 1999) (Nardis et al., 2005) (Ormos, 2002) (Ormos and Daragó, 2005).

Generally in solving (seismic) inverse problems the forward problem solution plays a central role; calculation of traveltime data is really computer time consuming. Usually reasonable approximations are to be applied that could give acceptably precise results and shorter running time. This idea is used also in this refraction inversion method but it caused some assumptions and approximations to be considered (Ormos and Daragó, 2005). Firstly high frequency limit is assumed, this way parameters of the structure might vary only slowly compared to the wavelength, so the radius of boundaries' curvature has to be significantly higher than the horizon depth. Besides this only those rays are considered in the calculations that propagate along the layer boundaries, so there is no penetration (diving) effect. Finally ray traces within each layer are approximated with straight lines, even in layers where the velocity is changing. But with these approximations the running time of the program turns to be very short (e. g. with 20 iterations, 780 observed data and 17 unknowns the running time is approx. only 45-50 seconds).

This method was tested and applied both on synthetic and field data and it was proved that it is usable in practice. However the question arises: what kind of errors this ray tracing approximation can cause in solving the inverse problem. Obviously errors could be caused by 2 reasons. Firstly it is probable in the context of the complexity of the model and the traveltimes computed in a simpler way that the more complex is the model the less correct the solution will be. In case of models with rapid changes in their structure worse results are expected but its measure is unknown. In this paper the precision of the usability of the method was investigated in case of geological models with different variability.

The Inversion Method

The firstly used method in inversion of refraction time data using function description was developed by Bernabini et al. (1988) who have calculated the coefficients of fourth degree power functions for describing the refractors of a multi-layered geological model, with laterally unvarying velocities.

Knowing the geological structures found in nature an inversion method has to be developed that is capable of treating slow lateral changes both in thicknesses and velocities (Ormos, 2002). This method was applied successfully for refracted data. With the help of this method determination of refracting horizons and layer velocities are possible. Lateral changes in both wave propagation velocities in the media and layer thicknesses are described by series expansions using adequately chosen basic functions. The $p_i(x)$ physical parameters of the investigated model (e.g. thicknesses and velocities) can be defined as follows (1).

$$p_i(x) = \sum_{j=1}^{Ji} C_{ij} \cdot F_{ij}(x) \qquad (1)$$

In the equation i represents the layer numbers, J_i the number of functions defining the i^{th} parameter, $F_{ij}(x)$ the j^{th} basic function of the i^{th} parameter and C_{ij} the j^{th} expansion coefficient of the i^{th} parameter. The basic function $F_{ij}(x)$ in case of trigonometric series takes the form of $sin(j*2\pi x_i/X)$ and $cos(j*2\pi x_i/X)$ (X means here the length of the profile). C_{ij}-s do not depend on the distance x along the profile and therefore they are suitable for determination (estimation) by inversion.

In solving the direct problem we have to state that some assumptions and approximations have to be taken mentioned in the introduction. Taking into consideration that the wavelength determined by the highest dominant frequencies attainable in investigation of near-surface velocity conditions the restrictions above are acceptable from the viewpoint of the application. As it was proved in case of

geoelectric application it is the trigonometric function series that is more suitable for describing geological structures and the number of coefficients as well (Gyulai et al., 2007).
In the solution of the inverse problem the estimation of the C_{ij} coefficients has to be carried out. After solving the inverse problem it is necessary to calculate the physical parameters of the 2D model from the estimated C_{ij} coefficients along the profile. The coefficients of the functions are calculated by a qualified linearized LSQ inversion, according to which the nonlinear problem can only be solved iteratively. The above described inverse problem is formulated in the following well-known form (2):

$$\vec{c} = \left(\underline{\underline{G}}^T \underline{\underline{G}} + \lambda \underline{\underline{I}}\right)^{-1} * \underline{\underline{G}}^T \vec{t} \qquad (2)$$

where $\vec{c}$ means the correction vector of the coefficients (compared to the previous estimation), $\vec{t}$ the vector of the differences between the observed and (from the estimated coefficients) calculated first arrival times, and $\underline{\underline{G}}$ the (Jacobi-) matrix of the partial derivatives of the traveltimes according to the C_{ij} function coefficients. $\underline{\underline{I}}$ represents the unit matrix, and the scalar λ is the damping factor.
The aimed investigations are qualified with the help of the following standards. The relative deviation D in the data space is defined as follows (3):

$$D = \sqrt{\frac{1}{I}\sum_{i=1}^{I}\left(\frac{T_i^{(observed)} - T_i^{(calculated)}}{T_i^{(calculated)}}\right)^2} \qquad (3)$$

(where $i = 1,\ldots,I$ denotes the number of time data).
The relative deviation in the model space which can be calculated from the values of $p_n(x)$ physical model parameters at m seismic source points can be calculated in the following way:

$$d = \sqrt{\frac{1}{M}\frac{1}{(2N-1)}\sum_{m=1}^{M}\sum_{n=1}^{2N-1}\left(\frac{p_n(x_m)^{(exact)} - p_n(x_m)^{(estimated)}}{p_n(x_m)^{(exact)}}\right)^2} \qquad (4)$$

(where m=1,…, M denotes the number of source points, x_m the distance of the m^{th} source point along the profile, n=1,…, 2N-1 the number of model parameters). In case of field measurements the relative deviation between the estimated and starting model parameters can be calculated in the similar form as in equation (4).

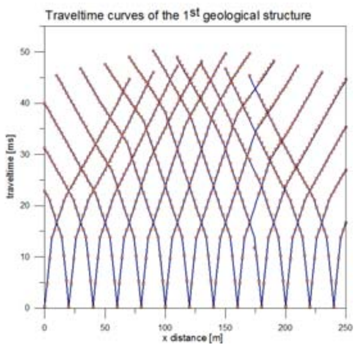
Investigations on Different Synthetic Geological Structures

For obtaining reliable results many geological structures have to be investigated. In this paper three different kinds of geological models are represented, all of which are 3-layered models with one horizontal layer near the surface and another layer boundary defined by a certain function. In the used geological models the second layer boundary was a syncline with a smooth shape (first model), a kind of step function (third model) and some synclinal shape with straight lines resulting the structure of the second model somewhere between the first and the third ones. The second and the third model contains rapid changes in the layer boundaries, the third one definitely does not suit to the condition system of the inversion method written above. To be able to separate the effects of the velocity and the layer thickness in this consideration the velocities are constant in each layer.
The common points in the investigated geological structures were the lengths in x direction (250 m), the depths (20 m), the number of sources (13; with 20 m distance between two of them), the number of geophones (126; with 2 m distance between two ones) and the propagation velocities in certain layers (400 m/s, 1400 m/s and 2400 m/s). This investigation was carried out using correct synthetic data computed with FD-Vidale type ray tracing, a method based on FD approximation of the eikonal

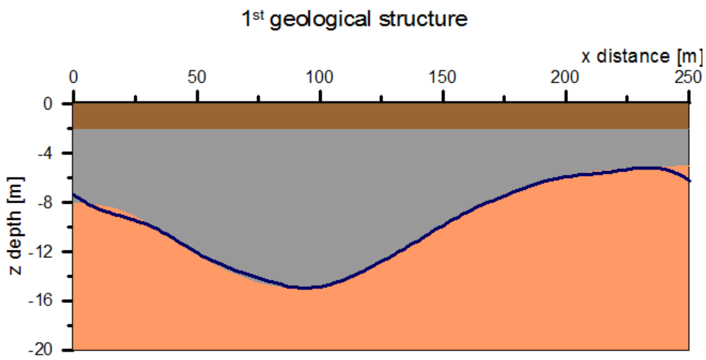
equation for calculating synthetic traveltimes using the ReflexW software by Sandmeier (Vidale, 1988). With this method transmitted, diffracted and head waves are also taken into account. This way the complexity of the model is not limited at all. Only the first arrivals can be calculated, so the method is only suitable to simulate picked refraction traveltimes (Sandmeier, 2006).
The calculated synthetic data were used in the inversion method as quasi "measured data" without additional error. For each model 783 data were used for the inversion, in which 20 iterations were performed for each model. As a target model the firstly given synthetic models were set and this way the difference or similarity between the initial and the calculated geological structure became easily definable. It is thought that the difference between the method correct in every aspect of wave propagation and the simple ray tracing (used in the inversion) appears in the difference between the initial and the computed models.

Results, Conclusions

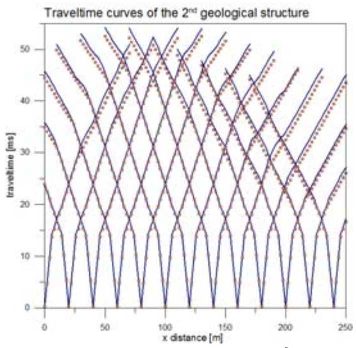
In the inversion process 17 coefficients (unknowns) were used, number of data was 783 in each case. As results of the inversion traveltime data are also got from the estimated coefficients (*Fig. 1.*). Here traveltime curves of each model can be seen, those of the initial (brown dots) and the estimated models (dark blue continuous lines) compared to each other. After the inversion the calculated geological structures can be drawn; the comparison of the initial models and the gained ones can be seen in *Fig. 2*. In these figures the layers of the initial model are shown in different colours and the calculated layer boundary between the second and the third layer is shown as a dark blue continuous line.
In case of the first structure, the model distance d (Eq.4) was 0.8 %, the data variance D (Eq.3) was 0.2 %. These mean really good fitting between the initial and the calculated models (*Fig. 2a*). In case of the second structure d = 1.6 % and D = 1.4 % (*Fig. 2b*). Investigating the third geological structure the results were just worse than those of the other two models mentioned before (*Fig. 2c*). Although the difference between the initial and calculated layer boundaries is just eye-catching, the model distance was only 8.7 % and the data variance was also just 3.7 %. The main characteristics of the structure can be recognized, but the straight-like structure turned to be a sinus-like line.



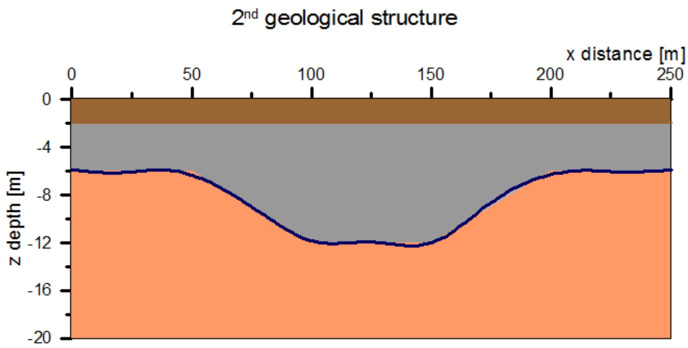
***1a)** Traveltime curves of the 1st geological structure.*



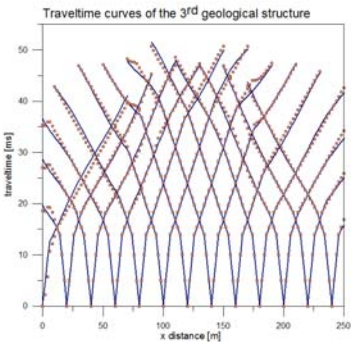
***2a)** Results of the 1st geological structure.*



***1b)** Traveltime curves of the 2nd geological structure.*

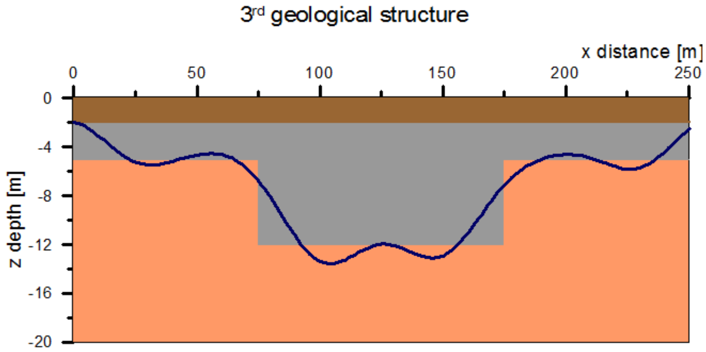


***2b)** Results of the 2nd geological structure.*



***1c)** Traveltime curves of the 3rd geological structure.*

***Figure 1** Comparison of the traveltime curves of initial and calculated geologic structures.*



***2c)** Results of the 3rd geological structure.*

***Figure 2** Comparison of initial and calculated geologic structures.*

These results proved that the investigated inversion method deals with shallow, elongated structures with relatively slow changes in the layer boundaries. But if there are steep layer boundaries or step-like changes in its depth the given results are less reliable, but not deniable. So as a conclusion such a simple forward problem solution found in our inversion method can calculate elongated geological structures that are relatively common in nature. Although in case of more complicated geological models using the FD forward problem solution is inevitable, similar to geoelectric cases (Kavanda et al., 2006). Further development of the method is planned in this direction.

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