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The Strategy of Joint Inversion Using Function Series

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SUMMARY

The paper presents the results of investigations on the choosing of the optimal number of coefficients (unknown) in the function inversion. Using synthetic noisy geoelectric data we show, that it can be find an optimum number of these coefficient using in the joint inversion. The method can be used also in case of field data the optimum of the coefficient's number is at the minimum of the main model parameter estimation error and the relative data distance as a function of the number of expansion coefficients.

INTRODUCTION

To investigate laterally „quickly" varying 2D and/or 3D near surface geological structures with geoelectrics a large number of inversion methods have been developed lately. The inversion methods for 2D and 3D geoelectric structures using discretisation according a regular grid are accurate, but they need a large number of unknown parameters to be determined. Therefore the estimation error of the unknown is high. Possibilities to improve reliability are application of joint or simultaneous inversion methods, reduction of the estimated parameters' number, or using of constraining methods. (Auken et al. 2006, Dobróka and Turai 2001, Dobróka 2004, Gyulai , Ormos and Dresen 2000, Kis et al. 2001, Loke and Barker 1996, Ormos and Daragó 2005, Turai, Dobróka and Vass 2006, Pellerin and Wannamaker 2005, Vozoff and Jupp 1975).For solution the problems mentioned above in this paper we use the widely applied function inversion method. This method using basic functions expanded in series to describe the physical and geometrical model parameters *p(x)*.However the questions are open: what kind of basic function is choosing, and how "long" (how many unknown) must be the function series to recognise an optimal solution in inversion. Due to work out the proper strategy of function inversion in this paper we have give an answer with results of investigation with synthetic geoelectric data.

DIRECT PROBLEM

For the direct problem we use to describe the thicknesses and apparent resistivities (i.e. the model parameters *p(x))* basic functions expanded in series. The direct problem is formulated as:

$$p(x) = \sum_{j=1}^{J} C_j F_j(x)$$

In the equation J indicates the number of elements of series, $F_j(x)$ the j^{th} basic function member and C_j the j^{th} coefficient. C_j-s do not depend on the distance x along the profile and therefore they are suitable for estimation by inversion. The direct problem can be solved with (local) 1D forward modelling, and with 2D/3D forward modelling (Spitzer 1995).

THE COMBINED INVERSE SOLUTION

The inverse problem means estimation of the C_j coefficients in the functions from the observed apparent resistivity data, using other parameters of measurement (number and places of electrodes) and a priori information (number of layers, number of coefficients). After solving the inverse problem, the 2D model's physical parameters *p(x)* are calculated from the estimated coefficients C_j along the profile, at arbitrary point. The nonlinear inverse problem has been solved with a linearized, iterative method using the L_2 norm (LSQ). We introduced the combined inversion where we use the 1D (i.e. 1.5D inversion) and 2D (i.e. 2D inversion) forward modelling sequentially to enhance the accuracy and reliability and reduce the computing time (Kavanda, Gyulai and Ormos 2006). The iteration process of combined inversion of our investigation is shown on Figure 1.

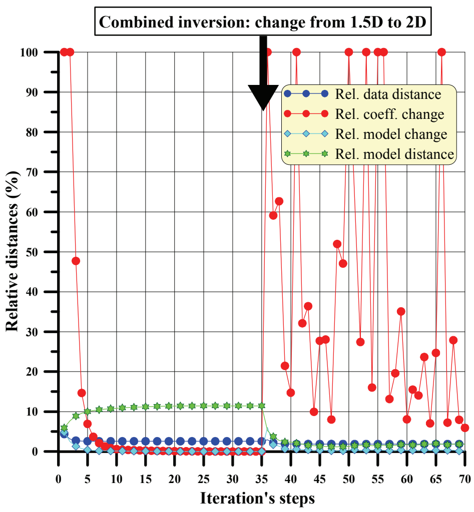

Fig.1: Iteration of combined inversion using noisy synthetic geoelectric data.

RELIABILITY OF THE ESTIMATED MODEL

From the covariance of the estimated coefficients and form the basic functions the error $\sigma_k(x)$ of the k^{th} physical model parameters can be calculated at arbitrary point x along the profile. To characterize the reliability of the inversion's results related to the whole model the mean estimation error $\overline{\sigma}$ is computed from the parameter's $\sigma_k(x)$.

$$\sigma_k(x) = \frac{\sqrt{\sum_{i=1}^{J(k)} \sum_{j=1}^{J(k)} \left\{ F_{kij}(x) * F_{kji}(x) \ * COV_{ij} \right\}}}{p_k(x)} * 100\% \qquad \overline{\sigma} = \sqrt{\frac{1}{K} \sum_{k=1}^{K} \sigma_k^2(x)}$$

We consider the solution of the (combined) inverse problem at the minimum of mean estimation error of the parameters, and at the minimum of the relative data distance D and relative model distance d (d only in case of synthetic model) **simultaneously**. (Salát et al. 1982, Menke 1984, Gyulai and Ormos 1999).

$$D = \sqrt{\frac{1}{I} \sum_{i=1}^{I} \left(\frac{\rho_i^{(observed)} - \rho_i^{(calculated)}}{\rho_i^{(calculated)}} \right)^2} \qquad d = \sqrt{\frac{1}{M} \frac{1}{(2N-1)} \sum_{m=1}^{M} \sum_{n=1}^{2N-1} \left(\frac{p_{mn}^{exact} - p_{mx}^{estamited}}{p_{mn}^{exact}} \right)^2}$$

DETERMINATION STRATEGY OF THE NUMBER OF COEFFICIENTS

The question is that can be giving a general solution in the inversion: how to choose the number of coefficients, and can be finding an optimum in the number of coefficients. The number of coefficients is depending on the geological structure and the noise; but the geological structure will be only known after the inversion. To solve this conflict we show a possible solution. For this investigation we choose a three-layer 2D model with straight line boundaries, shown on figure 2 (drawn with white colour). The layers resistivities are lateral no variable and have the values 10, 5, 20 ohmm respectively. We simulated (with 2D forward modelling) VES measurements on 29 stations at the profile with current electrode spacing (AB/2) between 1.6 and 50m. At least we added 2% random (Gauss) noise to the synthetic data. In the combined inversion we describe the layer resistivities with 1-1 unknown respectively. The layer thicknesses are described with trigonometrical series. In case of the first layer the number of coefficients are 3. For the second layer thickness we choose variable number of coefficients from 3 to 29 (Fig 2. white boxes). In case of each number of coefficients we made a combined inversion. (The iteration one of them is shown on figure 1.)

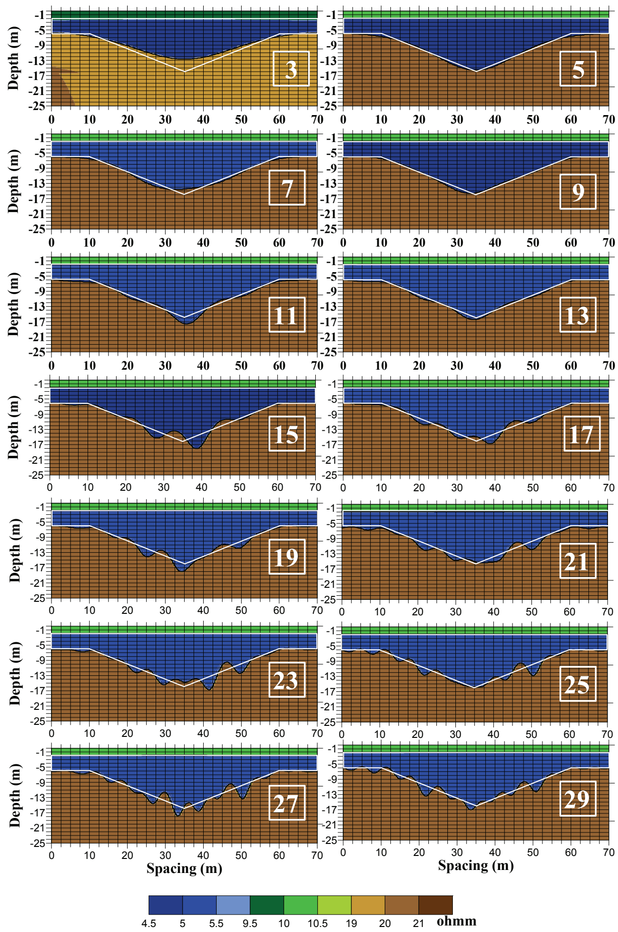

Fig.2: Resulting models of the inversion as a function of the coefficient's number

The resulting models are shown on figure 2. With increasing number of coefficients we are able to determine more detailed geological model, but with increasing in relative model distance. This effect is only in case of noisy data. The best fits on geometrical parameters can we see in case of coefficient's number 5 or 7 or 9.

More exactly results can be seeing on the Figure 3. The solution of the inversion is given at the minimum of the relative data distance. If only this parameter would be treated, all the solutions would be acceptable with more then 7 coefficients.

The relative model distance – related to the whole model – is given as a function of the coefficient's number of the second layer. The minimum value of the fitted curve give the optimum of coefficient's number.

In case of field data we don't know the exact model; but we can't calculate the relative model distance d. But we can calculate the mean estimation error: $\overline{\sigma}$. (Gyulai, Ormos 1999). On the figure 3a and 3b are shown, that the curves d and $\overline{\sigma}$ are in very good correlation.

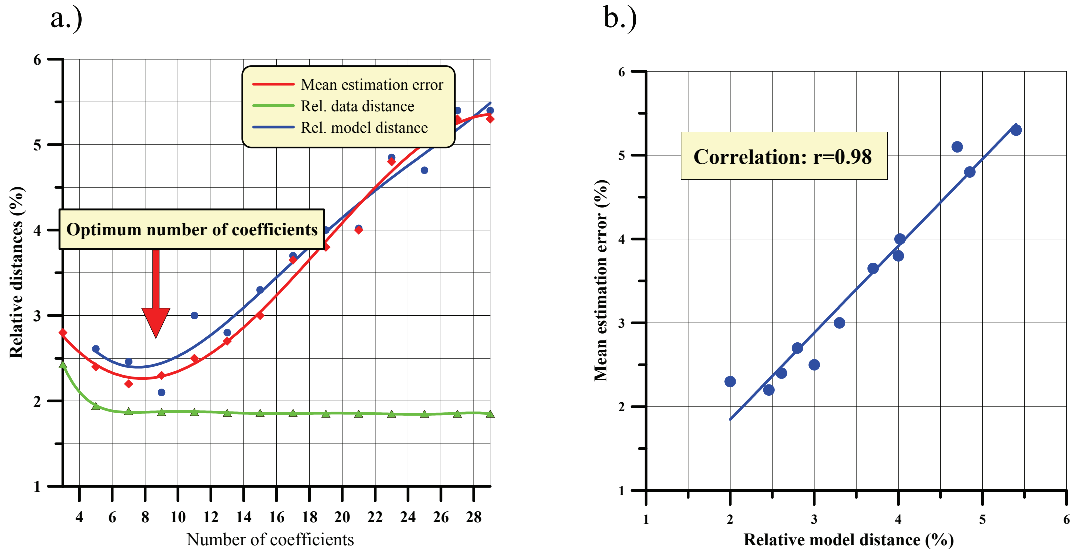

Fig.3: Determine the optimal coefficient's number, with correlation

Therefore in case of field data the optimum of the coefficient's number is given at the minimum of the relative data distance and at the minimum of the mean estimation error simultaneously.

CONCLUSION

In this paper we presented a method on the choosing of the optimal number of coefficients (unknown) in joint function inversion. We have shown with synthetic noisy geoelectric data, that the optimum number of the function coefficient is determined at the minimum of the relative data distance and at the minimum of the relative model distance. In case of field data we can use the mean estimation model parameter error instead the relative model distance. If we choose a higher number as the optimum, we can recognize the model more detailed, but with decreasing reliability (i.e. increasing error).

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