REFERENCES


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The solution of OQ 1156

Kramer Alpár-Vajk

In [1] the following sequence \((a_n)_{n \in N}\) is defined: \(a_1 := 3\) and further, for all \(n \in N; \ n \geq 2; \ a_n := \) the smallest number with \(a_{n-1}\) divisors.

According to the author of [1] the first six terms are 3; 4; 6; 12; 72; 559872: It is conjectured that \(\forall n \in N^* ; \ a_n + 1\) is prime.

The first observation is that the fifth term above is false because not 72 is the smallest number with 12 divisors but 60: In light of this, the next term is 5040 and since

5041 = 71 · 71 the conjecture is wrong.

REFERENCE


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The solution of OQ 1141

Kramer Alpár-Vajk

The subject of OQ 1141, see [1] is to prove that the sequence \((a_n)_{n \in \mathbb{N}}\) is finite.
This sequence is defined in the following way: \(a_1 := 1\); and \(\forall n \in \mathbb{N}; a_n :=\) the smallest natural number such that for all \(k \in \mathbb{N}; k < n; a_n - a_k\) is a prime or a power of a prime. We have the first six terms: 1,3,5,8,10,12. We will show that there is no other term in this sequence, supposing the opposite and distinguishing two cases.

Case 1. Suppose that \(x_7\) exists and is even. Then

\[\{x_7 - 8, x_7 - 10, x_7 - 12\}\]
are all even and in the same time one of them is divisible by 3, thus divisible by 6 and so neither a prime nor a power of a prime.

Case 2. Suppose that \(x_7\) exists and is odd. Then

\[\{x_7 - 1, x_7 - 3, x_7 - 5\}\]
are all even and in the same time one of them is divisible by 3, thus divisible by 6 and so neither a prime nor a power of a prime.

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A logarithmic equation (OQ 19)

Gabriel T. Prăjitura and Tsvetomira Radeva

ABSTRACT. We gave a solution to the Open Question 19.

MAIN RESULT

The Open Question 19 ([1]) asked for all \( n \) such that

\[ \log_2 3 + \log_3 4 + \ldots + \log_n (n + 1) = n + 1 \]

Equivalently, we are looking for all \( n \) such that

\[ n + 1 \leq \log_2 3 + \log_3 4 + \ldots + \log_n (n + 1) < n + 2 \]

Let \( p \) be a natural number. Notice first that if

\[ \log_2 3 + \log_3 4 + \ldots + \log_{n_0} (n_0 + 1) < n_0 + p \]

then

\[ \log_2 3 + \log_3 4 + \ldots + \log_n (n + 1) < n + p \]

for all \( n \leq n_0 \), while if

\[ \log_2 3 + \log_3 4 + \ldots + \log_{n_0} (n_0 + 1) \geq n_0 + p \]

then

\[ \log_2 3 + \log_3 4 + \ldots + \log_n (n + 1) \geq n + p \]

for all \( n \geq n_0 \).

This is because

\[
\sum_{k=2}^{n+1} \log_k (k + 1) - \sum_{k=2}^{n} \log_k (k + 1) = \log_{n+1} (n + 2) > 1 = (n + 1 + p) - (n + p)
\]

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Therefore, in order to solve the double inequality above we need to find two numbers \( n_1 \) and \( n_2 \) such that \( n_1 < n_2 \) and

\[
\log_2 3 + \log_3 4 + \ldots + \log_{n_1} (n_1 + 1) \geq n_1 + 1
\]

\[
\log_2 3 + \log_3 4 + \ldots + \log_{n_1 - 1} (n_1) < n_1
\]

\[
\log_2 3 + \log_3 4 + \ldots + \log_{n_2} (n_2 + 1) < n_2 + 2
\]

\[
\log_2 3 + \log_3 4 + \ldots + \log_{n_2 + 1} (n_2 + 2) \geq n_2 + 3
\]

When the two numbers are found, the solution is \( n_1 \leq n \leq n_2 \).

Next we will show that \( n_1 = 70 \). We must show that

\[
\sum_{k=2}^{69} \log_k (k + 1) < 70
\]

and

\[
\sum_{k=2}^{70} \log_k (k + 1) > 71
\]

which follows easily from the computation

\[
\sum_{k=2}^{69} \log_k (k + 1) = 69.998 \text{ and } \sum_{k=2}^{70} \log_k (k + 1) = 71.001
\]

Now we will show that . We must show that

\[
\sum_{k=2}^{105,555} \log_k (k + 1) < 105,557
\]

and

\[
\sum_{k=2}^{105,556} \log_k (k + 1) < 105,558
\]

which follows easily from the computation
\[
\sum_{k=2}^{105,555} \log_k (k + 1) < 105,556.9999955755
\]
and
\[
\sum_{k=2}^{105,556} \log_k (k + 1) = 105,558.0000037657
\]
Therefore
\[
[\log_2 3 + \log_3 4 + \ldots + \log_n (n + 1)] = n + 1
\]
if and only if \(70 \leq n \leq 105,555\).
We will end with some comments about the problem.
The series
\[
\sum_{n=2}^{\infty} (\log_n (n + 1) - 1)
\]
is divergent. To see this notice that
\[
\log_n (n + 1) - 1 = \frac{\ln (n + 1)}{\ln n} - 1 = \frac{\ln (n + 1) - \ln n}{\ln n}
\]
By the Mean Value Theorem applied to the function \(\ln x\) on the interval \([n, n+1]\), there is \(k_n \in (n, n+1)\) such that
\[
\ln (n + 1) - \ln n = \frac{\ln (n + 1) - \ln n}{(n + 1) - n} = \frac{1}{k_n} > \frac{1}{n + 1}
\]
Therefore
\[
\log_n (n + 1) - 1 > \frac{1}{(n + 1) \ln n}
\]
and since
\[
\sum_{n=1}^{\infty} \frac{1}{(n + 1) \ln n}
\]
is a well known divergent series, the Comparison Test implies the divergence of the series we considered above. One of the consequences of this fact is that for every \(k \geq 1\) the equation
\[ \log_3 3 + \log_3 4 + \ldots + \log_3 (n + 1) = n + k \]

has only a finite number of solutions. From our computation here it actually follows that there are solutions for every \( k \geq 0 \). To find the exact number of these solutions turns out to be a very difficult technical problem since, as we showed above, for \( k = 1 \) we already need 7 decimals of accuracy.

REFERENCE


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