CONSENSUS OF HEAT EXCHANGER
IN COAL-FIRED POWER PLANT

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SUMMARY

The paper presents a calculation methodology of condensing heat exchanger flue gases-water, used for waste heat recovery from exhaust gases in coal fired power unit. Computations based on Colburn-Hougen’s mathematical model. There was performed a modification of this model, basing on Mollier’s diagram, involving the addition of formulas that calculates value of flue gases humidity ratio at given temperature. Based on above model modification, authors worked out an example of computations of heat exchanger installed in flue gases duct in lignite power plant.

I. INTRODUCTION

The problem of waste heat recovery from exhaust flue gases in power engineering is an interest issue due to increase in power plant efficiency associated with waste heat utilization [13]. In last two decades, commercial technology of waste heat recovery from flue gases began to develop and the best methods of its utilization have been performed [12]. Today, there are no new coal fired power plants without units of waste heat recovery from flue gases [11]. According to authors knowledge, in no coal-fired units flue gases temperature is lowered below dew point. For greater heat flux recovery, there is a need to install heat exchanger with condensation of water vapor contained in exhaust gases.

Modeling of heat transfer process with condensation of water vapor contained in flue gases is complicated by the fact that condensation takes place in presence of inert gases. In the present case these gases are CO₂, CO, SO₂, O₂ and N₂. Additionally, heat transfer process consists of: heat flow between flue gases having temperature T_g and condensate with interface temperature T_i (Fig.1), and also steam mass diffusion due to difference in steam concentration in bulk y_H₂O and at the interface y_i.

The purpose of article is to present a mathematical model of condensing heat exchanger, used for waste heat recovery from flue gases in lignite power plant. This model is one-dimensional and bases on modification of Colburn-Hougen model (C-H) of condensing heat exchanger, which was described in [10]. In this paper the shell-and-tube heat exchanger was developed. Authors used some formulas from models [3, 7, 8, 10]. It was assumed that overall heat transfer process consists of two sub-processes: flue gases cooling without condensation of water vapor until steam dew point T_dew occurs and flue gases cooling with condensation. In opposite

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to [7, 8], Mollier’s diagram was added to calculate humidity ratio of flue gases X and condensate mass flux q_{mk}. In relation to [8], authors created another method for Lewis number calculation. Till now, in literature, a constant value of Lewis number was assumed [1, 8]. Thus Chapman-Enskog theory, having a limit to calculate mass diffusion coefficient D for two-component mixtures, was extended by introducing the hypothesis of proportionality of Lewis number Le to molar fractions of flue gases.

2. MATHEMATICAL MODEL

2.1. ASSUMPTIONS
Following assumptions were utilized in C-H model computations:
- heat flux is steady and one-dimensional;
- heat exchanger is shell-and-tube, countercurrent;
- flue gases are flowing across smooth copper tubes with cooling water inside;
- filmwise condensation of steam present in flue gases happens only when temperature of flue gases is equal to steam dew point T_{dew}. Condensation takes place only on surface of tubes;
- flue gases cooling process takes place in constant pressure p;
- cooling water mass flow in tubes \( \dot{m}_w \) is constant;
- heat losses from heat exchanger to environment are equal to zero.

One section of condensing heat exchanger is shown in Fig. 1.

![Fig. 1. Control volume with heat-flow parameters used in computations for one cell of heat exchanger](image)

2.2. MODEL OF HEAT TRANSFER WITHOUT CONDENSATION
Process of cooling flue gases without condensation continues until temperature of gases \( T_g \) reaches dew point \( T_{dew} \). Mass flow of water vapor \( \dot{m}_{H2O} \) and also mass flow of wet flue gases \( \dot{m}_{spm} \) don’t change. The process takes pace with constant humidity ratio of flue gases X.
Overall heat flux \( \dot{Q} \) transferred from flue gases to water consists only sensible heat from dry flue gases and water vapor:

\[
\dot{Q} = \dot{m}_{\text{spm}} \cdot \left[ c_{\text{p, spm}}(T_{\text{g, in}}) \cdot T_{\text{g, in}} - c_{\text{p, spm}}(T_{\text{dew}}) \cdot T_{\text{dew}} \right]
\]  

(1)

Differential equation of heat balance for water and flue gases is as follows:

\[
\alpha_g \cdot (T_g - T_i) \cdot dA = \dot{m}_w \cdot c_{\text{pw}}(T_w) \cdot dT_w = \alpha_w \cdot (T_i - T_w) \cdot dA
\]  

(2)

Heat transfer coefficient from flue gases \( \alpha_g \) was determined from calculation formula for Nusselt number \( \text{Nu} \) for pipe not covered with condensate layer [6]:

\[
\text{Nu} = 0.27 \cdot \text{Re}^{0.63} \cdot \text{Pr}^{0.36}
\]  

(3)

2.3. MODEL OF HEAT TRANSFER WITH CONDENSATION

Since the occurrence of condensation, water vapor mass flow \( \dot{m}_{\text{H2O}} \) and, at the same time, mass flow of wet flue gases \( \dot{m}_{\text{spm}} \) decrease lengthwise of heat exchanger according to mass balance equation:

\[
\dot{m}_{\text{spm}} = \dot{m}_g + \dot{m}_{\text{H2O}} = \dot{m}_g (X+1)
\]  

(4)

Colburn-Hougen equation is here of the form [5, 6]:

\[
\alpha_g \cdot (T_g - T_i) + r_{\text{H2O}} \cdot k_m \cdot (y_{\text{H2O}} - y_i) = k \cdot (T_g - T_w) = \dot{m}_w \cdot c_{\text{pw}} \cdot \frac{dT_w}{dA}
\]  

(5)

Humidity ratio is, in addition to temperature and pressure, the third parameter of state of flue gases. It is defined as the ratio of water vapor mass flow to mass flow of inert gases [7]

\[
X = \frac{\dot{m}_{\text{H2O}}}{\dot{m}_g} = \frac{M_{\text{H2O}}}{M_g} \cdot \frac{p_{\text{H2O}}}{p - p_{\text{H2O}}},
\]  

(6)

where partial pressure of water vapor \( p_{\text{H2O}} \) in function of flue gases temperature \( T_g \) is determined by approximation formula [3]:

\[
p_{\text{H2O}}(T_g) = 61 \cdot \exp(7.257 \cdot 10^{-2} \cdot T_g - 2.937 \cdot 10^{-4} \cdot T_g^2 + 9.81 \cdot 10^{-7} \cdot T_g^3 - 1.901 \cdot 10^{-9} \cdot T_g^4)
\]  

(7)

Then condensate mass flux is equal to

\[
\dot{m}_k = \dot{m}_g \cdot (X_i - X_2)
\]  

(8)

So water steam molar fraction in bulk \( y_{\text{H2O}} \) is

\[
y_{\text{H2O}} = \frac{\frac{\dot{m}_{\text{H2O}}}{M_{\text{H2O}}} \cdot \frac{\dot{m}_{\text{H2O}}}{\dot{m}_{\text{H2O}} + \dot{m}_g} \cdot \left( 1 - \frac{\dot{m}_{\text{H2O}}}{M_{\text{H2O}} \cdot (\dot{m}_{\text{H2O}} + \dot{m}_g) \cdot M_{\text{H2O}}} \right)}{M_{\text{H2O}} \cdot \frac{\dot{m}_{\text{H2O}}}{M_{\text{H2O}} + \dot{m}_g}}
\]  

(9)

According to above formula, Colburn-Hougen equation can be solved by algebraic method, without using of successive approximations method. In this equation, there is exactly one unknown – condensate interface temperature \( T_i \).
Saturation temperature drop, determined by approximation formula [3], due to condensing water vapor and decreasing steam partial pressure $p_{H_2O}$, influences on molar fraction of uncondensed steam at the interface (Antoine equation [1, 5]):

$$y_i = 1000 \cdot \exp \left[ \frac{16,262 - 3799,89}{T_{dew} + 226,35} \right] \frac{p_{atm}}{\nu}$$  \hspace{1cm} (10)

Heat transfer coefficient from flue gases $\alpha_g$ was calculated by formula [2]:

$$\alpha_g = \frac{11,36}{3} \left( \frac{\lambda_k \rho_k (\rho_k - \rho_{H_2O}) g \nu_{H_2O}}{4 \mu_k (T_{dew} - T_w) b} \right)^{0,08} \left( \frac{y^2_s \rho_s \rho_{g,n} \alpha \mu}{g \lambda_k \rho_k} \right)^{0,08}$$  \hspace{1cm} (11)

In this mathematical model authors also developed a procedure for calculating flue gases Lewis number directly from its definition. According to Chapman-Enskog’s theory [4], the value of diffusion coefficient of water vapor in one flue gases component is equal to

$$D_{H_2O}(D; T; T_g) = D(T) \cdot \left( \frac{T_g + 273,15}{T} \right)^2$$  \hspace{1cm} (12)

where $D(T)$ – diffusion coefficient of water vapor in each flue gases component in temperature $T$, given in Tables [7].

The above formula is limited to binary mixtures. Thus, for exhaust gases, being a multi-component mixture, there was proposed a hypothesis of proportionality of diffusion coefficient for molar fractions of each gaseous component. According to this hypothesis, formula for flue gases Lewis number is as follows:

$$Le = \frac{y_{CO2} \cdot D_{H_2O}(0,202 \cdot 10^{-4}; 307,4; T_g)}{v_{spm}(T_g) \cdot (y_{CO2} + y_{N2} + y_{O2})} + \frac{y_{N2} \cdot D_{H_2O}(0,293 \cdot 10^{-4}; 298,2; T_g)}{v_{spm}(T_g) \cdot (y_{CO2} + y_{N2} + y_{O2})} + \frac{y_{O2} \cdot D_{H_2O}(0,282 \cdot 10^{-4}; 308,1; T_g)}{v_{spm}(T_g) \cdot (y_{CO2} + y_{N2} + y_{O2})} \cdot Pr^{-1}$$  \hspace{1cm} (13)

Flue gases Lewis number value is necessary to determine the mass transfer coefficient [6]:

$$k_m = \frac{\alpha_g \cdot M_{H_2O} \cdot \ln \left( \frac{1 - y_i}{1 - y_{H_2O}} \right)}{c_{pg} \cdot M_{spm} \cdot (y_{H_2O} - y_i) \cdot Le^{\frac{3}{2}}}$$  \hspace{1cm} (14)

Overall heat flux $Q$ transferred to water is composed of: sensible heat flux from dry flue gases, sensible heat flux from water steam, latent heat flux and heat flux from cooling condensate. Then

$$Q = q_{mpm,in} \cdot c_{p,spm}(T_{dew}) \cdot T_{dew} - q_{mpm,out} \cdot c_{p,spm}(T_{g,out}) \cdot T_{g,out} + \rho_{H_2O} \cdot q_{mk} + q_{mk} \cdot c_{p,w} \cdot (T_{dew} - T_{g,out})$$  \hspace{1cm} (15)

In above formulas, physical properties of fluids (flue gases, vapor, condensate and cooling water) were calculated with utilization of approximation formulas [3].

An algorithm of computations of heat exchanger, basing on above modifications, was described in [11].
3. RESULTS AND DISCUSSION

Fig. 2 shows heat exchanger geometry and assumed parameters.

Other input data for calculations is the flue gases temperature drop in each cell \( \Delta T_g = 2 \) K. Mole fractions of gaseous components for lignite are: \( y_{CO2} = 0.1211, y_{SO2} = 0.0025, y_{O2} = 0.0266, y_{N2} = 0.5943, y_{H2O} = 0.2484 \). Assumed mass flow of flue gases corresponds to 50% of the exhaust gases mass flow in lignite fired power unit with power equal to 900 MWel.

Total area of heat transfer surface \( A \) was determined as a result of numerical integration of expression \( \frac{dA}{dT_w} \), meaning area of heat transfer surface needed to heat water by 1 K, as a function of water temperature \( T_w \) (Fig. 3).

Fig. 4 presents the dependence of heat transfer surface \( A \) on cooling water temperature \( T_w \) decrease. This graph is general solution of differential equations (2) and (5). Overall heat transfer area, which was determined by numerical calculations, equals \( A = 35834.8 \) m\(^2\).

Fig. 2. Assumptions for example

Fig. 3. Increase of area of heat transfer surface \( \frac{dA}{dT_w} \) in dependence on cooling water temperature \( T_w \)

Fig. 4. The dependence of heat transfer surface \( A \) on cooling water temperature \( T_w \) decrease.
To validate the algorithm of computations, a comparison of heat exchanger calculations was performed by using presented algorithm (Tab. 1) and classical Colburn-Hougen method (Tab. 2).

**Tab. 1. Results of heat exchanger computations by modified Colburn-Hougen method**

<table>
<thead>
<tr>
<th>Cell</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Final results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_g, ^\circ C$</td>
<td>65,1</td>
<td>62</td>
<td>60</td>
<td>58</td>
<td>56</td>
<td>$\Sigma q_{mk} = 30,37$ kg/s</td>
</tr>
<tr>
<td>$T_w, ^\circ C$</td>
<td>59,2</td>
<td>40,2</td>
<td>32,7</td>
<td>26</td>
<td>20</td>
<td>$\Sigma Q = 77,3$ MW</td>
</tr>
<tr>
<td>$q_{mk},$ kg/s</td>
<td>–</td>
<td>9,07</td>
<td>8</td>
<td>7,05</td>
<td>6,25</td>
<td>\begin{align*} A &amp;= 7011,65 \text{ m}^2 \ L &amp;= 2,88 \text{ m}\end{align*}</td>
</tr>
<tr>
<td>$Q,$ MW</td>
<td>–</td>
<td>22,9</td>
<td>20,3</td>
<td>18</td>
<td>16,1</td>
<td></td>
</tr>
<tr>
<td>$dA/dT_w,$ m$^2$/K</td>
<td>439,7</td>
<td>138,88</td>
<td>117,7</td>
<td>105,9</td>
<td>96,2</td>
<td></td>
</tr>
</tbody>
</table>

**Tab. 2. Results of heat exchanger computations by classical Colburn-Hougen method**

<table>
<thead>
<tr>
<th>Cell</th>
<th>0</th>
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<th>2</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$T_g, ^\circ C$</td>
<td>65,1</td>
<td>62</td>
<td>60</td>
<td>58</td>
<td>56</td>
<td>$\Sigma q_{mk} = 30,1$ kg/s</td>
</tr>
<tr>
<td>$T_w, ^\circ C$</td>
<td>59,2</td>
<td>41</td>
<td>31,7</td>
<td>24,6</td>
<td>20,1</td>
<td>$\Sigma Q = 76,7$ MW</td>
</tr>
<tr>
<td>$q_{mk},$ kg/s</td>
<td>–</td>
<td>7,9</td>
<td>10</td>
<td>7,5</td>
<td>4,7</td>
<td>\begin{align*} A &amp;= 6987,58 \text{ m}^2 \ L &amp;= 2,9 \text{ m}\end{align*}</td>
</tr>
<tr>
<td>$Q,$ MW</td>
<td>–</td>
<td>20,1</td>
<td>25,2</td>
<td>19,1</td>
<td>12,3</td>
<td></td>
</tr>
<tr>
<td>$dA/dT_w,$ m$^2$/K</td>
<td>439,7</td>
<td>143,6</td>
<td>114,3</td>
<td>102,2</td>
<td>96,3</td>
<td></td>
</tr>
</tbody>
</table>

Error analysis of two calculation methods was also conducted. Several comparative variables were selected for each computational cell of heat exchanger section with condensation: water temperature $T_w$, condensate mass flow $q_{mk}$, heat flux $Q$ and increase of heat transfer area $dA/dT_w$. Finally: total mass flow of condensate, overall thermal power, heat transfer surface area $A$ and heat exchanger length $L$ were compared.

The largest calculation uncertainty (about 6%) was obtained in result of condensate mass flow $q_{mk}$ determination in each cell. This discrepancy is due to the fact that in computations by classical Colburn-Hougen method, water vapor partial pressure in each cell of heat exchanger is assumed, not calculated.

Computations error in final results is not greater than 1%, which leads to the conclusion about correctness of developed algorithm.

### 4. CONCLUDING REMARKS

1. Preformed computations confirmed that the proposed modification of Colburn-Hougen method allows for solving Colburn-Hougen equation by algebraic method. It highly simplified heat exchanger calculations.
2. Calculations of flue gases Lewis number by proposed method gave values of this parameter in range of 0.7÷0.75. This calculated values are similar to those constant values assumed in literature references.

3. Based on numerical computations of heat exchanger part with condensation by using classical and modified Colburn-Hougen method, it was found a good agreement between values calculated by both methods. It allowed confirmed the correctness of developed algorithm of computations.

REFERENCES


[7] JEONGK.: Condensation of water vapor and sulfuric acid in boiler flue gas, Rozprawa doktorska, Lehigh University, 2009


