Abstract: Product designers should pay attention to many attributes of a product during design, however, studies have shown that besides the functionality, the aesthetically-pleasing appearance of products attracts consumers, and it has a significant effect on the buying process. Thus, it is efficient to know what geometries, colours, materials, etc., are pleasing to the human eye. This article deals with geometries that appear in nature since these forms are pleasing to many people of different generations and cultures and provide a sense of harmony. The hypothesis is that the usage of the mathematically describable principles of nature – such as Fractals, Golden Ratio, Fibonacci Sequence, Voronoi Diagram – could result in a better-looking design that increases the success of the product, like in the cases of Stradivarius violin, Aston Martin, or Apple logo.

Keywords: Product design, Nature-inspired design, Mathematics in design, Geometry, Aesthetics

1. Introduction

Nowadays, there are many design methods and principles for product designers. Designers should pay attention to the product life cycle, the sustainable material choice, the user experience, the practical and user-friendly design, etc. That is why design education is about to face interdisciplinary challenges [1, 2], where continuous support and development is necessary.

There is no one rule of successful design, however, there are some fascinating facts about the most successful products, which could show that besides many factors, beauty and harmony have a massive impact on us, as Donald Norman said: “Attractive things work better.” [3, 4] Other studies [5–9] have also shown that the factor of beauty or aesthetics is an essential criterion of the quality of design.

Forms appearing in nature are aesthetically-pleasing to the human eye and provide satisfaction and harmony by their proportion. That is why some researchers already tried to gain inspiration from nature while designing objects [10]. The hypothesis is that if designers implement the beauty of nature in their design, it could bring more success. This article introduces examples of the forms in nature with mathematical
explanations and connected examples of arts since art and design have the same goal by attracting consumer attention. Although art and design could be affected by nature and mathematical objects on many levels, this article deals with Fractals, Golden Ratio, Fibonacci Sequence and Voronoi Diagram. Besides that, nature forms inspired product examples introduced in this paper prove that the usage of these geometries can be principles in the field of design and could bring more success for designers and engineering designers.

2. **Fractals**

Fractal is a self-similar geometry that develops through iterations. The name “fractal” originates from the “fractus” Latin word, which means broken, unsmooth. It was introduced by Benoit Mandelbrot in 1976, however, the mathematical history of fractal goes back to the 19 century. One of the main milestones of this journey was in 1904 when Helge von Koch defined the Koch curve. In Figure 1, a few iterations can be seen, where the process begins with a unit length line. At the first iteration, the center is extracted and replaced with two lines with 1/3 length, at 60° angle. The process is continuing with an infinite number of iterations [11, 12].

![Figure 1](image)

**Koch curve at iterations from 0 to 3**

With fractal, Mandelbrot could describe natural phenomena.

“Why is geometry often described as ‘cold’ and ‘dry’? One reason lies in its inability to describe the shape of a cloud, a mountain, a coastline, or a tree. Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line. Nature exhibits not simply a higher degree but an altogether different level of complexity,” Benoit Mandelbrot [13].

*Figure 2* presents a few examples of fractal appearance in nature. Among others, fractal — the self-similar geometry — can be discovered in peacock feathers, lake (on the collage: Lake Nasser, Egypt) tentacle, eyes, cauliflower, and leaves as well.
In the 1980s – parallel to technical innovations – fractal geometry became displayable, which opened up a new world for both mathematicians and artists. Among the branches of fine arts, we may already speak of a separate fractal art where software creators are able to produce similar images shown in Figure 3 with software support.
Fractals – among others – are appearing on clothing design, website design, and architecture as well. In order to show some architectural examples, in Figure 4 a ceiling in Iran [17], the Sky Habitat for Singapore designed by Moshe Safdies [18], and the wall of the Centenary of the Federation of Australian States [11] can be seen.

3. GOLDEN RATIO

The origin of fractal geometry is derived from ancient Greek philosophers, just like other ratios-systems. Among one of them is the Golden Ratio that is widely considered to be the most perfect and beautiful proportion. The Golden Ratio (Figure 5) means that the longer segment “a” is proportional to the shorter segment “b”, at the same rate, the sum of “a” and “b” is proportional to the “a” segment [19]. If “a” the longer segment and “b” is called the shorter, the proportion may be described by:

$$
\frac{a}{b} = \frac{a+b}{a} = 1.6180339887 \text{(approx.)} \tag{1}
$$

Figure 4
Fractal in architecture: Ceiling in Iran [17],
the Sky Habitat for Singapore designed by Moshe Safdies [18],
the wall of the Centenary of the Federation of Australian States [11]

Figure 5
Explanation of Golden Ratio
Golden Ratio can be found in nature, in architecture, in design, in engineering, in arts as well [20–22]. Golden Ratio is appearing of the human body, for instance, on the human face (length of face/width of face, length of mouth/width of nose and so on), on the human hand (e.g., the proportion of the first two to the full length of the finger), on human lungs, on the hearing and balance organ, on the cardias cycle and even on the brain waves [23]. Figure 6 presents a few proportions of the human body.

![Figure 6](image)

**Figure 6**
Golden proportion of human body

Since Golden Ratio is very pleasing for the eye and provides harmony, it is commonly used in the design of art. Figure 7 shows an example of Leonardo Da Vinci’s pictures, The Last Supper. The dimensions of the room were created by the usage of the Golden Ratio.

![Figure 7](image)

**Figure 7**
The Last Supper based on Golden Ratio
In Figure 8, a Golden Ratio inspired violin can be seen. It was designed by Antonio Stradivari, which is the most valuable string-playing instrument in the world because of its harmonic qualities [24, 25]. (It is sold for nearly 16M dollar.)

Golden Ratio also found in car designs such as Volkswagen Beetle [20] or Aston Martin [26] as in Figure 9. The Aston Martin DB9 is described as:

“Perfectly Proportioned – Every inch of DB9’s form is designed for elegance and balance. The simple beauty of nature guides the design of DB9, with the ‘golden ratio’ setting all proportions. The result is a profile where every line, dimension and proportion works in harmony. Combine this with the near perfect weight distribution, provided by a lower engine placement, and you have a DB9 balanced on sight and in experience.” [26, 27]
4. **Fibonacci Sequence**

A similar arithmetic approach is the Fibonacci Sequence, named after the Italian mathematician Leonardo Fibonacci. Fibonacci Sequence: Starting with the beginning of the sequence, we inscribe a series of numbers where the next member is always composed of the sum of the two preceding members. The sequence of the numbers is: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, … etc. [28, 29](Figure 10). The quotient of the series is approaching to the number of golden ratio: ~1.618...

![Figure 10](image)

**Explanation of Fibonacci Sequence**

This sequence can be seen widely in the natural world, for example, in seahorse, plants, flowers, storms, snail, and so on (Figure 11).

![Figure 11](image)

*Figure 11*

_Nature creation examples of Fibonacci Sequence: seahorse [30], carnivorous plant [31], sunflower [32], hurricane [33], calla lily [34], cuban snail [35]*
For an example of art, a rhythm of Béla Bartók can be mentioned (Figure 12). Many composers used Fibonacci numbers and Golden Ratio in their composition in order to provide a formal structure of them [36].

![Figure 12](image)

*Figure 12*

*A palindromic rhythm based on Fibonacci numbers in the opening of the third movement by Béla Bartók [36]*

As shown in Figure 13, the Apple logo is based on the Fibonacci Sequence. It is designed by Paul Rand, who is an American graphic designer with a successful career in the field of logos [37]. (He has references in IBM, UPS, Enron, Westinghouse, ABC, and Steve Jobs’ NeXT as well.)

![Figure 13](image)

*Figure 13*

*Apple logo, based on the Fibonacci Sequence*

5. **VORONOI DIAGRAM**

Voronoi Diagram is an algorithmic diagram that was formulated by a Ukrainian mathematician Georgy Voronoy (1868–1908). (The concept was only published in the year of his death [38].)

When introducing the Voronoi Diagram, we use the concept of distance. In this article, we will work with the Euclidean distance. Denote the Euclidean distance between points \( p(p_x, p_y) \) and \( q(q_x, q_y) \) by \( d(p,q) \).

Then

\[
d(p,q) = \sqrt{(p_x - q_x)^2 + (p_y - q_y)^2}
\]
Let $P = \{p_1, p_2, \ldots, p_n\}$ be a set containing $n$ different points in the plane. The Voronoi diagram of $P$ is interpreted as the division of the plane into $n$ cells (ranges), with the property that a point $q$ belongs to a cell belonging to point $p_i$ if and only if

$$d(q, p_i) < d(q, p_j) \quad \forall p_j \in P, j = i$$

(3)

Denote the Voronoi diagram of $P$: $\text{Vor}(P)$.

The cell belonging to $p_i$ is denoted by $V(i)$ and is called the Voronoi cell of $p_i$.

Based on the interpretation of the Voronoi Diagram, this cell has the property that the points inside it, among the points $P$ are closest to $p_i$ [39, 40].

The cells can be formed by section bisector set on lines drawn between the points (Figure 14a, b, c). A plane shape is the set of points closest to the examined point from the available points (Figure 14d). The cells of the Voronoi mesh are complex polygons in the plane (Figure 14e) or complex polyhedra in space.

**Figure 14**

Explanation of the Voronoi Diagram

This mathematical model appears as natural patterns in many phenomena. To show some examples, in Figure 15, we can see a honeycomb, giraffe skin, sea turtle, dry earth, dragonfly wing, and soap bubble.

**Figure 15**

Voronoi Diagram appearance in nature: honeycomb [41], giraffe skin [42], sea turtle [43], dry earth [44], dragonfly wing [45], and soap bubble [46]
The Voronoi Diagram is also used in many cases in art. For example, it is possible to create mosaic images or portraits by its help as illustrated in Figure 16.

![Figure 16](image1.jpg)

**Figure 16**
Application of Voronoi Diagrams in Arts: An example of an automatic method for mosaicing images by using Voronoi diagrams: the input image (left) and the result image (right) [47]; Portrait called Segmentation and Symptom by Golan Lenin [48]

The application of the Voronoi Diagram is widespread among architects and product designers. For instance, Figure 17 shows the 2011 Millennium Yacht Design Award winner, the extravagant 125-meter superyacht called Voronoi created by Kim Hyun-Seok, a South Korean industrial architect. The designer said that this lattice exterior provides harmony with its natural appearance [49]. Figure 17 also shows the Voronoi Chair designed by Franken Architekten. The cellular form of wood is based on the Voronoi Diagram. The chair is created from a cluster of spatial cells that behave similarly to the Voronoi cell structure. The designer writes that the cellular covers establish a feeling of balanced proportions [50].

![Figure 17](image2.jpg)

**Figure 17**
Voronoi Diagram inspired superyacht [51] and chair [50]

6. **Conclusion**

With the development of science, scientists could describe the phenomena of nature better and better. This knowledge is useful for product designers as well, not only because mathematical concepts are not far from design concepts (such as symmetry, pattern and so on), but also the beauty and harmony of nature can be implemented in the design, as it is shown in case of many successful products in the past.
These examples have proved that if designers consider these principles during the concept-development of the product design process, the solution could bring more success.

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