JOVAN KARAMATA (1901-1967)

THE INTELECTUAL BIOGRAPHY OF AN MATHEMATICIAN

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*This paper presents the intellectual biography of one of the most renowned Serbian mathematician, Jovan Karamata. His family and his life way are depicted in detail. His most significant results in the field of classical mathematical analysis, more precisely in Tauberian theorems and in the theory of summability in general, are given. These results, as well as those related to slowly varying functions, inequalities, trigonometrical integrals, have frequently been quoted in various papers and monographs. For history of Serbian mathematics there are equally significant the great number of his students, later eminent mathematicians recognized as "Karamata's (Yugoslav) school of mathematics".*

*Keywords: Jovan Karamata, University of Belgrade, Serbian Academy of Sciences and Art, Tauberian theorems, Regularly varying functions.*

the FAMILY ROOTS

Jovan Karamata comes from an old, respectable and wealthy family, originally from Katranitsa (today Pyrgoi, in the vicinity of Mavrovo of the Eordaia municipality in North Macedonia of modern Greece), which was among the first Greek and Aromanian merchants to move to Zemun in the middle of the 18th century. At that time, the Zemun Greek-Aromanian colony already had over seventy families, mostly from Moscopole and Katranitsa. At the beginning of the 19th century, the colony numbered about eight hundred souls, and by the end of the century, very few remained. Many proud and famous Greek-Aromanian families had disappeared. However, members of the Karamata family adapted very quickly and successfully to the new environment, establishing family ties and partnerships with the Serbs. The first generations of Karamatas, born in Zemun, had already rejected the Greek-Aromanian-Turkish traditional dress and customs, and adopted all the traditions of the Serbian civil society and French-Austrian culture at the turn of the 18th and 19th centuries. They often educated their children abroad, chose a family patron saint, entertained Serbian writers and moved in upper-society circles with the most prominent Serbs in Vojvodina.

During the Austrian rule in northern Serbia, from 1718 to 1739, the Karamatas, in accordance with tradition, trade cattle and build stables in the Morava Valley. In the middle of the 18th century, following the Treaty of Belgrade and the Austrian retreat along the Sava and Danube, they cross into Austrian territory and settle in Srem. Now even wealthier, they expand their trade activities and build stables in Bežanija near Zemun, and in 1772, Dimitrije Karamata and his wife Marija Grujić, a Serb from Constantinople (Istanbul), purchase a large, grand one-story house not far from St. Nicholas Church in Zemun. This house, the home of the Karamata family for eight generations, was built in 1763 and today, it represents a valuable architectural and artistic monument.

Dimitrije Karamata had three sons - Anastas, Jovan and Filip - who branched out the pig trade. In Serbia, occupied by the Ottomans, they bought lean pigs that were fed in their own stables in the Morava Valley and Bežanija. After they were fattened, they were taken as far as Italy and Austria, but mostly to Sopron in northwestern Hungary, to a famous cattle market at that time. The extent of the pig trade within Austria and later the Austro-Hungarian monarchy is evidenced by the fact that the Karamatas, along with two other Zemun families, owned their stables even in Eperjes on the Hungarian-Austrian border, and that they applied to the Viennese authorities for permission to build an Orthodox chapel for their religious needs. By dealing in this trade, the Karamatas increased their material wealth and Dimitrije commissioned Georgije Tenecki, one of the most famous Serbian portrait painters of the 18th century, to do his portrait, as well as a portrait of his wife Marija and minor son, Teodor. However, they did not forget their origin, and so Anastas was among the founders of the first Greek school in Zemun in 1794. The social reputation that the family enjoyed in Zemun is evidenced by the fact that at the time of the siege of Belgrade, during the Austro-Turkish War in 1788, the Austrian Crown Prince Francis, later Emperor Francis II, stayed in their home with his uncle, the Austrian Emperor Joseph. II. But, the Serbian character of the family is expressed in the first half of the 19th century, when Vuk Karadžić, Branko Radičević and other Serbian writers, poets and cultural representatives were guests in the Karamata family home.

Dimitrije's son, Jovan Karamata (1780-1846), had three children: two sons, Marko, who was Mihail Obrenović's secretary for a while before he became ruler, and Atanasije, and a daughter, Julijana, who was married to the director of the Zemun quarantine hospital.

Atanasije Karamata (1821-1868), our Jovan's grandfather, was the most significant Karamata of the 19th century. He graduated from the Vienna Polytechnic, where he studied trade and technical sciences, and after the founding of Serbian Vojvodina in July 1848, he was appointed to manage its finances, as can be seen by the "assignat" dated August 13 – signed on the right as "chairman" is Jovan Šupljikac and on the left "as cashier" Atanasije Karamata. In the fall of 1848, Serbian Patriarch Josif Rajačić, and the clerical, political and military leader of the Vojvodina Serbs at the time, stayed in the Karamata family home, together with the Main Committee for the Fight against Hungarian Domination. At the end of the revolt, Atanasije was engaged in trade and was a member of the "town council". He was one of the originators of the *Zemunski glasnik* newspaper and was considered a man of noble heart with a passion for public affairs.

Atanasije and his wife Marija, born Jovica, had four children - Kosta (Konstantin), Jovan, Stevan - the father of our mathematician Jovan - and Ozren.

From generation to generation, the family increased its wealth. But, perhaps at the peak of their financial prosperity, in the 1860s, the swine fever devastated their barns, and phylloxera destroyed their vineyards, which came at about the same time as the world economic crisis. At the time when these disasters shook the family's finances, Jovan's grandfather, Atanasije, also passed away, so the trustees put the family assets in order. The pig trade was liquidated, a good part of the vineyards was sold, and Atanasije's widow, Marija, was left with an old family house.

Stevan Karamata (1866-1940), Jovan's father, graduated from a merchant school and since he was very good at his job, the wealthier Serbian merchants invited him to join them in establishing the Serbian Bank in Zagreb. The Bank developed rapidly, becoming the second largest in Croatia (after the First Croatian Savings Bank), and soon a branch was established in Budapest (Podunavska Bank), with Stevan as its director. However, neither he, nor the Karamatas were owners of the voting capital in the Serbian Bank, but "only" one of the shareholders, and a capable and hardworking well-to-do bourgeois family whose one member, Stevan, was a member of the management board of the Bank. Finally, it should be emphasized that the surname Karamata was officially introduced by him, as according to the tombstone in Gardoš in Zemun, Stevan's ancestors were called Karamat i.e. without the final letter a.

In this setting, bourgeois and wealthy, both materially and spiritually – because we must not forget that Jovan's uncle Kosta Karamata (1862-1920) was a gymnasium mathematics teacher in Zemun, respected not only as a teacher but also as a scientist (he published three papers in the RAD journal of the Yugoslav Academy of Sciences and Arts in Zagreb), and uncle Jovan, in addition to being the owner of a famous printing house, was the director of the savings bank in Zemun and a member of the Hungarian Parliament – Jovan Karamata was born in Zagreb on February 1, 1902 as the sixth child (the first was Atanasije, who died as a child, and then came Ozren, Srđan, Smiljka and Kosta (Konstantin)) of Stevan and Desanka, born Vukomanović.

Education and Introduction to Science (1908-1928)

By a combination of circumstances, Jovan Karamata became acquainted with the sometimes-harsh reality at an early age. He started attending primary school in Budapest in 1908, which he continued and finished in Zemun in 1912. At nine years old, he lost his mother, the prematurely deceased Desanka Karamata (1870-1910). "Nanna" Marija, the widow of the late grandfather Atanasije, took it upon herself to care for the children. The devoted and deeply pious woman, about whom Jovan Karamata spoke with great respect, had, according to him, a lasting influence on his spiritual development. In 1912, he enrolled in the Zemun Gymnasium, but because Zemun was located in the turbulent and dangerous border area between Austria-Hungary and Serbia, in 1914, just before the start of World War I, he left Zemun and continued his education in Osijek (1914/15) and then in Sušak near Rijeka, cities that were still under the rule of Austria-Hungary. Moving from place to place seems to have been very difficult for him, both because of having to leave his parents' home and Zemun, the place he always loved and considered his home, so he was close to repeating the third year of school. Finally, worried about the war and the general uncertainty, the father decides to take his children, Srđan, Smiljka, Kosta and Jovan, to Switzerland. Ozren joined them only after the withdrawal of the Serbian army across Albania, at the beginning of 1917. After a one-year stay at a boarding school in Cressier (Canton of Bern), where he studied French, among other things, Jovan moved to Lausanne where, in 1916, he enrolled in a gymnasium (Collège). He was completely alone during this time. He lived in Vidy Park, in the Fischer boarding house, today the headquarters of the International Olympic Committee. In 1918, he successfully graduated and moved on to Gymnase Scientifique, which he attended with Marko Čelebonović, later a famous Serbian painter, and Georges De Rahm, a future colleague in the Department of Mathematics at the University of Geneva. He graduated in 1920.

Jovan had acquired a solid knowledge of mathematics at the gymnasium. He always attracted the attention of the physics professor, who was surprised when he experimentally determined and calculated the boundaries of the refraction of light. This was not anything new at the time, but he did it completely on his own. Since he chose to study natural sciences, he did not have the opportunity to get to know better and take a liking to literature, painting or music. However, what undoubtedly played the most significant role in his future was that he had acquired the virtues of Swiss Protestantism - precision, meticulousness, systematicity and diligence - which were clearly manifested in his life, and especially his scientific works.

After graduating, Karamata returned to his homeland which, by that time, had experienced radical changes. Belgrade had become the capital of a great state. In addition to its political significance, it was now becoming an increasingly attractive economic and financial center. His father, Stevan Karamata, moved to Belgrade, where he ran the newly established Jadransko-Podunavska Banka and quickly gained a good social reputation. Among other things, he was appointed Honorary Consul to the Kingdom of Denmark and represented banking at the Rotary Club. The fact that Milan Stojadinović, Minister of Finance of the Kingdom of Serbs, Croats and Slovenes, later Yugoslavia, often consulted with him also speaks of the reputation he enjoyed in the field of finance. Ideal working conditions were created for the young Karamata. Wide prospects opened up before him.

The country was rising from the rubble of war and there was a great need for civil engineers, so on the advice of his father, in 1920 Jovan Karamata enrolled in the Department of Civil Engineering at the Faculty of Technical Sciences of the University of Belgrade. After the preparatory exam in descriptive geometry, mathematics, physics, mechanics and mechanical properties of materials, which was taken after the second year of study, Professor Bogdan Gavrilović recognized young Karamata’s talent for mathematics. He immediately recommended him to Mihailo Petrović, on whose advice Karamata, in 1922, transferred to study mathematics at the Faculty of Philosophy, University of Belgrade. There he attended lectures on the first group of sciences - theoretical mathematics, applied mathematics and experimental physics.

At that time, after the departure of Mladen Berić and Sima Marković, only Bogdan Gavrilović (Faculty of Engineering) and Mihailo Petrović (Faculty of Philosophy) remained at the University of Belgrade. They were also the only Serbian mathematicians who were members of the Serbian Royal Academy of Sciences, in addition to Petar Živković. The distinguished academician, Professor Milutin Milanković, excelled in applied mathematics. In the early twenties, two Russian immigrants, already prominent mathematicians, Anton Bilimović and Nikola Saltikov, as well as Radivoje Kašanin, one of the few Serbian mathematicians who did not come from Petrović’s Belgrade school of mathematics, came to Belgrade and the University.[[1]](#endnote-1)

Mihailo Petrović, who was correct in his assessment that without young people there can be no serious progress in new and modern fields of mathematics, began with the training of young associates. From the post-war generation of mathematicians, he chose Tadija Ž. Pejović, a substitute at the Second Belgrade Gymnasium for Boys, as his assistant. In addition to Gavrilović, Milanković, Bilimović, Saltikov, Petar Zajančkovski and Vjačeslav Žardecki, Kašanin, a young mathematician who studied outside Belgrade, graduated in 1921 in Paris, received his doctorate in Belgrade and later became a member of the Serbian Academy of Sciences, also took an active part in Petrović's Mathematical Seminar at the University of Belgrade. All of them, after studying at universities all over Europe, returned to Belgrade with a broad, excellent knowledge in various fields of mathematics.

The first results of such organized teaching and scientific work at the University and the Academy soon became evident. New generations of young scientists, mathematicians, began to exhibit an interest in the most modern branches of mathematics, and in that amalgam of both old and new concepts, approaches and methods, our mathematics was achieving great success. The atmosphere among mathematicians in Belgrade at the time was perhaps best described by academician R. Kašanin in his memoirs. He says: ''In addition to their higher education and original scientific papers, all three (M. Petrović, B. Gavrilović and M. Milanković, A/N) possessed something I value most, something I consider to be among the highest human values: a love for younger generations, an understanding of young people, selflessness and a sincere desire to help young, talented people in their progress. They were delighted and enjoyed seeing young people make headway. I was lucky to have had the chance to develop and work alongside these great authorities in science and ethics. To be proud of their friendship. I doubt that the atmosphere created by Gavrilović, Petrović and Milanković existed anywhere else.'' Among the youngest mathematicians, still students, Miloš Radojčić and Jovan Karamata stood out the most, and although they completely differed when it came to their temperament, life and mathematical aspirations, they were the greatest of friends all their life.

Karamata did not mind much for formal school knowledge, and even as a student he aspired to do independent research work. The first teacher, an example and role model in scientific work, a man he respected all his life, was Mihailo Petrović. According to the Serbian mathematician, academician Miodrag Tomić, he was the one who passed on to him the great and sincere love of science, the wide outlook, the desire to do scientific work, ideas free from formal restraints, and mathematical directions one should take in search of results. As a result, Karamata chose, as his field of work, the theory of functions by limiting processes, one of the areas in which Petrović gave elegant and significant results and which had already made Petrović's name known. According to the testimony of academician M. Tomić, Karamata himself often said that he was greatly influenced by his acquaintance, and later friendship, with Radivoje Kašanin. With his rich and extensive knowledge of mathematics, which he acquired at the universities of Vienna and Paris, Kašanin expanded the mathematical horizons of young Karamata. It enabled him, as Karamata himself admits, to see the importance of new areas of mathematics – set theory, measure theory and integral theory – and especially the importance of rigorous proof, characteristic of the German school of mathematics.

But, despite the undeniable influence of Petrović and Kašanin on the development of Karamata as a mathematician and a man, it is a known fact that Karamata was self-taught, that he was not a follower of any mathematical school and that he never had a renowned mathematical name that could pave the way for him to be in the company of top mathematicians. He succeeded completely independently thanks to the value of his works. And to achieve success on this path, one needs books and works of the greatest mathematicians as companions. Mathematical literature is Karamata’s third and perhaps most important teacher. As a student, living in a studio apartment in the attic of a building in the heart of Belgrade, he studied the textbook of H. Vogt, *Eléments de Mathématiques Supérieures* and the famous paper of Hermann Weyl, *Über die Gleichverteilung von Zahlen mod. Eins,* to the smallest detail. He got the inspiration and the idea for his dissertation by solving problems from the famous and, for the development of classical mathematical analysis in Belgrade, very important collection of problems and theorems by George Pólya and Gábor Szegö, *Aufgaben und Lehrsätze aus der Analysis*. Thanks to Edmund Landau's monograph, *Darstellung und Begründung einiger neuer Ergebnisse der Funktionentheorie*, Karamata encountered the Hardy–Littlewood tauberian theorem (Godfrey Harold Hardy, John Edensor Littlewood), which he immediately began to study and which will quickly earn him great recognition in world mathematics. Karamata once said: ” I Wanted to improve my knowledge of the foundations of the theory of functions. This is the reason why I first started to study the theory of series, but when I entered into this theory, I found so many old and new results that I remained there.” [[2]](#endnote-2)

In May 1925, after he graduated, he was appointed temporary assistant professor under Professor M. Petrović, and in January 1929 full-time assistant professor of mathematics at the Faculty of Philosophy, University of Belgrade. For a while, he worked as an intern at the Seismological Institute within the Faculty of Philosophy.

Jovan Karamata's first texts in mathematical literature, which appeared immediately after graduating in 1925, were review papers on the works of his professor Mihailo Petrović. His first public appearance was at the Serbian Royal Academy on December 14, 1925, at the age of 23, where he presented a scientific paper on his first work in the theory of series entitled *O izračunavanju granica vezanih za dvostruke nizove brojeva*, which was approved for publication in GLAS SKA, at a meeting of the Academy of Natural Sciences held on February 1, 1926. Mihailo Petrović and Bogdan Gavrilović commented positively on the discussion. The content of this paper was included in his doctoral dissertation, *O jednoj vrsti granica sličnih određenim integralima*, which was accepted for the final doctoral examination on March 9, 1926, at the session of the Faculty of Philosophy, University of Belgrade, according to the report of the examination board members: Mihailo Petrović, Nikola Saltikov and Anton Bilimović. On March 22, 1926, he passed the doctoral examination before the same examination board and was promoted to doctor of philosophy. In the legacy of Milutin Milanković, there is a letter from Mihailo Petrović sent to Professor Milanković on May 26, 1926 from Paris, in which he sent him a copy of *Nôte de Karamata*, which shows that not only was his presentation truly good, but so was „the idea in the dissertation".[[3]](#endnote-3)

His next work, *Sur Certaines Limites Rattachées aux Intégrales de Stieltjes*, was presented at the meeting of the Paris Academy of Sciences on March 29, 1926, in which he tied the results obtained in previous works to Stieltjes integrals, at which time the famous French mathematician Jacques Hadamard, referring also to the previously presented work of French mathematician Paul Levy, stated the following: ''Mr. Karamata's astute ideas partly coincide with some of those presented by Paul Levy. On the other hand, the two very different methods in which these two authors came to the solution, demonstrates the productivity of a new kind of thinking.'' The main theorem in this paper was reproduced in the famous monograph by Frédéric Riesz and Bela Szökefalvi Nagy, *Vorlesungen über Funktionalanalysis,* Berlin 1956.

In April 1926, Karamata went to serve in the military, where he remained for 12 months. He left the army with the rank of corporal because he was dismissed from military school as "incompetent". That is to say, on the officer exam, he listed all the lakes of the Kingdom of Serbs, Croats and Slovenes, but forgot to mention Lake Bled in Slovenia, where King Alexander often spent his summers[[4]](#endnote-4).

When Karamata was granted a scholarship from the Rockefeller Fund, he resided in Paris from December 1927 to September 1928. His specialization at the department of Henri Lebesgue gave him the opportunity to meet and even make lasting friendships with many both eminent and younger mathematicians, such as Paul Montel and André Lichnerowich. During his stay in Paris, he also participated in his first conference, the 52nd Congress of the French Association for the Advancement of Science (AFAS), held in July in La Rochelle.

The Golden Age (1929-1940)

The most significant and fruitful period of Karamata's creative work began after his return from Paris. In a relatively short period of time, from 1929 to 1933, he wrote his most famous and original works, which made his name known in almost all mathematical circles in Europe. He worked tirelessly, and the silent, night hours were ideal for him. He was given strong and selfless support and assistance from his wife Emilija Nikolajević (1906-1959), a lawyer employed at the State Attorney's Office, who was fluent in German and French, and was, in a way, her husband's secretary and "typist". For years, she had typed and retyped all his works. In addition, she participated in the extensive correspondence that Karamata had with mathematicians throughout Europe.

During this period, when his creative potential was at its peak, in addition to mathematics, Karamata also dedicated his time to anthroposophy, so he translated two lectures from 1909 of the founder of anthroposophy, Rudolf Steiner, *Praktično vaspitanje mišljenja* (Belgrade, 1929), for the Yugoslav Anthroposophical Society, and also wrote a short original text, *Upoznavanje sebe kao osnov saznanja*, which was published 1931 in the first issue of the journal for anthroposophy and art *Upoznaj sebe*. He often exchanged anthroposophical views on the world, life, science and creativity with his friends and fellow mathematicians Stanimir Fempl and, especially, Miloš Radojčić, with whom he continued to meet regularly for the rest of his life. Through anthroposophy, he also came in contact with art. He was a patron to a refugee Russian painter, Kravchenko, whom he introduced to the anthroposophical theme of paintings and pastel colors characteristic of anthroposophy. These additional activities did not prevent mathematics from becoming not only his love and occupation, but also his obsession and passion.

In order to have as much time as possible to think about mathematics and write papers, which were noticed immediately after publication and highly rated, without any interruption, he organized and held all his classes at the University in just one day, from eight in the morning to eight at night. He would have endless discussions with his student and associate, Vojislav Avakumović, about the mathematical problems he was working on intensively and feverishly, with almost superhuman energy. These problems dealt with the theory of series, more precisely the Tauberian theorem, named after the Austrian mathematician Alfred Tauber who, in 1897, proved the assertion that if the Abel sum of a series ∑ *an* exists and if the condition *nan* = *o(1)* applies, then it is also convergent. In 1910, Littlewood replaced Tauber's condition with a much more general *nan = O(1)*, and Hardy and Littlewood replaced it in 1914 with the condition *nan* ≤ *O(1)*.

Proof of Littlewood's assertion remained very complicated, despite the attempts of many mathematicians (Landau, Hardy, Robert Schmidt, etc.) to simplify it, until 1930, when Karamata's paper, *Über die Hardy-Littlewoodsche Umkehrungen des Abelschen Stätigkeitssatzes*, consisting of only two pages, appeared in the journal *Mathematische Zeitschrift*, and caused quite a stir in mathematical circles, bringing its author instant world fame. Professor Vojislav Marić’s comment on this occasion is interesting: ''During my visit to the University of St. Andrews in Scotland, I was introduced to the eminent mathematician Edward Thomas Copson from whose books many in my generation learned the theory of functions of a complex variable. Already quite old and not very interested in visitors, instead of the usual conventionalities, he only said: ‘So far, I’ve heard of only one Yugoslav mathematician – Jovan Karamata. When I was studying with Hardy in the 1930s, on one occasion, I found him walking around nervously in his office. Without greeting me, visibly excited, he said, “I received a letter from a young man from Belgrade who claims to have proved Hardy-Littlewood’s theorem on only two pages. It's just impossible!”’''. But it was possible! The first his original idea was that Abel summability of the series ∑ *an* to the sum *s* implied the same for any power *xn* and, consequently, for every polynomial of an arbitrary degree. The second idea was in the direct application of the Weierstrass theorem on uniform approximation of continuous function by the polynomials.

 

This Karamata's paper, which the editorial board of the *Mathematische Zeitschrift* included in its selection of the 50 most important papers among thousands that were published, on the occasion of its 60th anniversary, gave not only new, surprisingly simple and especially elegant proof of a well-known theorem, but also provided a new method that enabled many further solutions and applications, which as such, found its place in the well-known monographs of Karl Knopp *Theorie und Anwendung der unendlichen Reihen*, 1931; Gustav Doetsch, *Theorie und Anwendung der Laplace Transformation*, 1937; Edward Charles Titchmarsh, *The Theory of Functions*, 1939; David Vernon Widder, *The Laplace Transformation*, 1946; G. H. Hardy, *Divergent Series*, 1949; Jean Favard, *Cours d'Analyse*. *Compléments et Exercices d'Analyse*, 1962-1963. Here is how some of these authors saw Karamata's proof: "Previously known proofs of Littlewood's theorem were very complicated, in spite of the number of research devoted to it, till in 1930 J. Karamata found a surprisingly simple proof ", (Knopp) , "We shall give an extremely elegant proof which has recently been obtained by Karamata", (Titchmarsh), and Landau, in his reply to Mihailo Petrović, who had sent him Karamata's work with his proof, wrote that he regretted that he had not published his, already mentioned, monograph on the theory of functions a little later, because the chapter on inverse theorems would have looked different if he had known about this proof. Landau also sent the work to Knopp and asked for his opinion. Later on, when Knopp got better acquainted with Karamata, he told him: "When I saw what it was about, I was convinced that the proof was written by Landau himself and that, as a joke, he signed it using a Japanese name."

Although Norbert Wiener, the great American mathematician and originator of cybernetics, gave a general theory of inverse theorems in 1932, which contains most of the above-mentioned results, Karamata's method did not lose significance and was used in the proof of numerous new results. Wiener once said: "The elegant method of Karamata requires only one textbook theorem: the Weierstrass approximation theorem!" [[5]](#endnote-5). Wiener's method, as the strongest and most general, was also the most difficult because it was based on the deep theorems of the theory of Fourier transforms (Jean-Baptiste Joseph Fourier). According to academician Miodrag Tomić, Karamata sensed some sort of connection and the significance of the shape of the core of an integral transformation, but he was far from a complete theory.

In almost all his later works on the subject of Tauberian theorems, Karamata, in accordance with Wiener's theory, studied inverse theorems in integral form and with the most general conditions of convergence. He gave a number of variants and equivalents of those conditions, and the proof was often based on a new theory on which Karamata worked intensively in the late twenties and early thirties. These were the foundations of the theory of regularly and slowly varying functions. If the papers related to problems of theorems of Tauberian type immediately brought him fame and made his name known in the mathematical world, the true value of his papers on regularly and slowly varying functions was recognized only with the later development of mathematics, primarily the theory of probability.

In 1930, in addition to the already-mentioned paper on the functions of Tauberian type, his paper, *Sur un Mode de Croissance Régulière des Fonctions*, in which he gives the definition and basic properties of regularly and slowly varying functions, appeared in volume *III* of a minor Romanian journal, *Mathematica* (Cluj). The aim and idea of ​​this paper was to generalize Tauberian conditions in some inverse theorems of Tauberian type for the Laplace transform. He defined regularly varying function as a positive continuous function *r(x)* which satisfies , while for *ρ=0*, he defined slowly varying function *L(x)* if *.* Starting from these two simple definitions, Karamata developed the whole new theory of slowly and regularly varying functions which has included the majority of the most important properties of these functions. He also formulated two important theorems - on uniform convergence and on canonical representation.



Using both the concepts of regular varying functions and generalization of Abel's and Tauber's theorems in Laplace-Stiltjes transform, Karamata in 1931 obtained another famous result presented in the paper *Neuer Beweis und Verallgemeinerung der Tauberschen Sätze welche die Laplasche und Stieltjesche Transformationen betreffen[[6]](#endnote-6)*. That is the theorem, later and today frequently called Hardy-Littlewood-Karamata theorem, which was characterized as one of the most widely useful theorem in probabilistic (amongst other) context (E. Seneta) and as one of the major results in the analysis of the 20th century (N.H. Bingham). Here is this theorem:

*Let L(x) and L(1/x) be slowly varying functions, and let A(x) be a non-decreasing function on [a, ) such that the function converges for every x > 0. Than the following two statements hold:*

This theorem extends the Hardy-Littlewood's result for the Laplace transform where *L(X)=1*. E. Seneta said that this, the famous theorem of Karamata was one of the most famous and very widely useful theorems in probabilistic (amongst other) context[[7]](#endnote-7), and N. Bingham marked it as one of the major analytic results of this century[[8]](#endnote-8)!

In addition to Karamata, the "Yugoslav school" of mathematics, as E. Seneta and N. H. Bingham called it, consisting of his students and associates (Slobodan Aljančić, Bogdan Bajšanski, Ranko Bojanić, Miodrag Tomić, Vojislav Marić and others) played an important part in the further development of the theory of regularly varying functions. These results were not duly noticed or evaluated because, by comparing the extensive literature in today's most renowned publications on this topic, it is easy to see that only a small number of the more significant papers, written in the period from Karamata's introduction of the concept of regularly and slowly varying functions to its “reactivation” in the mid-1960s, were cited. Apart from Karamata and his students, practically the only papers mentioned were those written by Korevaar, van Aardenne-Ehrenfest, de Bruijn (1949), de Bruijn (1959) and Mtuszewska (1962, 1964, 1965). The testimony of N.H. Bingham: "One thing that puzzled me when I began work in this area was why Karamata and his co-workers had turned aside from this promising line of work in 1963. I asked Ranko Bojanić this when I met him at Ohio State University in 1988. He replied that ‘they hadn’t known what it was good for’! ".

This was the case until the famous monograph, *An Introduction to Probability Theory and its Applications*, written by William Feller, appeared in 1966, the second volume of which contains elements of Karamata's theory, but with not always precise assumptions and clear conditions. It soon became apparent that these functions could be successfully applied in many branches of mathematical analysis, the theory of differential equations and the theory of probability, wherever not only the fact of convergence but also other additional conditions were required. Thus, Karamata's theory, beyond all his expectations, grew into a huge mathematical building whose significance is still on the rise and which was, among other things, dealt with in the well-known monographs on the regular variability of functions – E. Seneta, *Regularly Varying Functions*, 1976; N. H. Bingham, C. M. Goldie, J. L. Teugels, *Regular Variation*, 1987; J. L. Geluk, L. de Haan, *Regular Variation Extensions and Tauberian Theorems*, 1987; J. Korevaar, *Tauberian Theory*, 2004 – and, in recent times, extended to functions of several variables. In these monographs, the theory of regularly varying functions and its application, compared to the original Karamata theory and its development until the beginning of the 1960s, is given in a modern, polished and significantly expanded form. While N. Bingham and E. Seneta wrote a lot about the role of regularly and slowly varying functions in probability theory together with historical overview of the main results, recently it is published in the journal "Historia Mathematica" a paper on the history of role and application of regularly varying functions in the theory of ordinary non-linear and linear differential equations. The main goal of this paper was to present an overview of the most influential achievements, focusing on those from the 20th century. Karamata's theory found its applications in partial and functional differential equations, difference equations and systems of differential equations, establishing the terms regularly varying functions and regularly varying sequences as important concepts in these areas. In the first half of the past century, special attention was given to differential equations of mathematical physics – the Thomas–Fermi (nonlinear) equation – and astrophysics and cosmology – the Emden–Fowler (nonlinear) and Friedmann (linear) equations. The mathematical aspect of these subjects was covered in detail in two notable mono-graphs. The first one by V. Marić, which could be considered the turning point in connecting the two fields of mathematical analysis in modern times and contains results up to the year 2000, and the second by Rehák, which contains more recent results up to 2014.[[9]](#endnote-9)

In this period of his life, Karamata had several almost equally remarkable and not less significant results in other fields of mathematical analysis and general mathematics. We mention some of them.

In the short paper *Sur inégalité relative aux functions convexes* (1932) using analogues Stieltjes integral inequality, Karamata formulated and proved the theorem on majorization inequality that generalizes the discrete form of Jensen's inequality from 1906. Although Karamata thought that Hardy and Littlewood had proved before him in 1929 the same theorem, what he added in his already published paper, this significant result was cited in two well-known monographs about inequalities by G.H. Hardy, J.E. Littlewood & G. Pólya (1934) and E.F. Beckenbach & R. Bellman (1961) as an original addition to Hardy-Littlewood's proof, but not as repeated proof. This inequality was named after Karamata and proved in detail.

A.P. Calderón and A. Zygmund in the well-known paper *On singular integral* (1956) refer to Karamata's paper *Ein Konvergensatz für trigonometriche Integrale* (1937) as significant for the beginning of the later development of a very complicated theory of trigonometric and singular integrals, as well as its natural continuation in form of important theory of singular integral operators and pseudo-differential operators which opened the new area of research in contemporary mathematical analysis.

We also mention the Karamata's paper *Théorèmes sur la sommabilité exponentielle et d'autres sommabilitiés s'y rattachant* (1932) which was continuation of examinations of various summability procedures. He formulated, as he called, Stirling procedure of summability, which he found as inversion of Borel summation of positive series. Late in the fifties and early in the sixties these results were studied more intensively and were extended, beside others, in papers by R.P. Egnue and by V. Vučković & B. Martić, who named such procedure of summation Karamata-Stirling.

Shortly after the publication of the papers on the theorems of Tauberian type and the theory of regularly varying functions, and based on the praises he received from some of the greatest mathematicians at the time, he was offered a professorship at the University of Zagreb. However, at the insistence of Belgrade mathematicians, led by Mihailo Petrović, Karamata remained in Belgrade, where on September 29, 1930, he was appointed assistant professor of mathematics at the Faculty of Philosophy. In the early thirties, when Karamata's teaching career at the University of Belgrade began, his colleagues at the Faculty of Philosophy and Technical Sciences had already significantly improved their methods of teaching mathematics by modernizing them. This was done gradually. Karamata, as a young teacher with high ambitions, but insufficient pedagogical experience and knowledge with regard to faculty reality, was the most radical in introducing new subjects and the way they were presented. He was very strict about exams and mid-terms, and it was quite difficult for students to pass his mathematics courses. The reason for this could also be that they had only one exam, which was held at the end of a two-year course, although it is also true that most students found it quite difficult to accept new scientific disciplines. New scientific concepts and new theories are often difficult, and an already established opinion does not easily accept new ideas. Karamata wanted to squeeze the old ideas out of the students' heads, and that could be met with resistance from many.[[10]](#endnote-10)

The first real recognition of his scientific work, in the former Yugoslavia, came when he was elected a corresponding member of the Yugoslav Academy of Sciences and Arts in Zagreb, on May 31, 1933. On February 20th of the same year, academician Dr. Vladimir Varićak submitted a written presentation on Karamata and proposal for his admission to the Academy. Thanks to his scientific reputation, in December 1935, Jovan Karamata was proposed for admission to the Czech Royal Scientific Society, and on January 8, 1936, he was elected its corresponding member. In May 1937, he was appointed associate professor at the Department of Theoretical Mathematics of the Faculty of Philosophy, University of Belgrade. Jovan Karamata gained great recognition all for his previous scientific and pedagogical work when he was elected a corresponding member of the Serbian Royal Academy at the main annual meeting of the Serbian Royal Academy, held on February 16, 1939. This was made official at the formal annual gathering held on March 7 of the same year.

By this time, he was already a member of the following scientific societies: Société Mathématique Suisse, Société Mathématique de France, Association Française pour L'avancement des Sciences and Deutsche Mathematiker Vereinigung, as well as a full-time correspondent for review journals *Zentralblatt für Mathematik und ihre Grenzseniebiete* (Giesen) and *Jahrbuch über die Fortschritte der Mathematik* (Berlin, Prussian Academy of Sciences). Karamata presented about 60 papers in those journals, including the papers of the most prominent mathematicians at the time.

Of course, during this ten-year period, from the late twenties to the beginning of World War II, Karamata participated in a large number of scientific meetings and congresses, where he was well received both as a lecturer and as a respected scientist. He was also a welcome guest at many universities across Europe. By 1939, in addition to the already-mentioned congress in La Rochelle, he participated in nine other congresses, where he always presented some of his results.

As a visiting professor, he lectured at universities in Poland (Lviv, Warsaw, and Poznań); Romania (Cernanz) in early 1933; Germany (Hamburg and Gottingen) in June 1936; Switzerland (Lausanne, Geneva); Belgium (Brussels - Institut des Hautes Etudes de Belgique); and again Germany (Stuttgart, Göttingen, Kiel, Berlin, Hamburg, Giessen, Leipzig, Jena) during 1937. At the invitation of the German minister of education, during the winter semester of the 1937-38 school year, he taught a special course in the theory of functions at the University of Tübingen.

He was also invited to give a series of lectures at the Sorbonne in Paris, several universities in India and to participate in the work of the Italian Academy of Sciences, Convegni Volta, but he was prevented from doing so by the outbreak of World War II. He had quite a reputation in the mathematical world!

In the mid-thirties, the zeal and enthusiasm from his youth slowly subsided. Although Karamata continued to work and publish his works in the most prestigious journals, he was somewhat dissatisfied. Thinking that these contributions do not represent any significant progress, contrary to his principles, he began putting aside unfinished works. However, this does not mean that he had begun to withdraw from Belgrade's mathematical life. Before the outbreak of the April War in 1941, he participated regularly in the sessions of the Academy of Natural Sciences, where, since 1939, together with M. Petrović, B. Gavrilović and A. Bilimović, he wrote seven reports on papers submitted for publication in GLAS SKA, the official journal of Serbian Royal Academy. During this period, he worked with Mihailo Petrović on arranging and systematizing Petrović's mathematical works. In the fall of 1940, he designed and was the first to conduct a course in the theory of probability at the Faculty of Philosophy, which lasted only one semester due to the outbreak of the April War in 1941. During this time, he was host to distinguished guests and great mathematicians like Paul Montel, Wilhelm Blaschke and Paul Erdős. Since he owned a car, he took the guests on trips across Yugoslavia, to their great satisfaction. The witty Blaschke later noticed that Karamata spent more time under the car than behind the wheel, "fixing breakdowns". Visits to Belgrade by these great mathematicians speak of the reputation that Belgrade mathematics enjoyed at that time. This is also evidenced by a lesser-known fact. Namely, Max Born, a renowned theoretical physicist, later winner of the Nobel Prize, feeling threatened by political changes in Germany, discussed with Karamata, in Göttingen, possible cooperation and a possible move to Belgrade.

Perhaps because he was weary of mathematics, or even because he subconsciously felt that the most productive years of creative work were coming to an end, Karamata gradually devoted more and more time and energy to activities outside mathematics and in the period from 1933 to 1935, he built his family house on Svetonikolajevska Street in Zemun. It was during this period that he and his wife Emilija had their four children: two sons, Vladimir (born in 1934, architect in Geneva) and Dimitrije (born in 1935, professor at the Medical Faculty of the University of Lausanne), and just before the war, twin daughters, Marija (1938-1940) and Katarina (1938-2005, biologist in Geneva). Since he studied mathematics mostly at night, Karamata always spent time with his family during the day. He renovated and furnished the house together with his wife. He loved spending time with his children and, from his numerous travels around Europe, he brought back a whole variety of anthroposophical and technical toys. He also led an intense social life and had a close relationship with his brothers Ozren and Kosta. Ozren, as the oldest male of the family, inherited the old family house. In order to preserve good family relations, the extended family would often gather in this house, and in hard times, relatives were always provided with a roof over their heads. Jovan Karamata and his family would spend summers with his brother Kosta, manager of an agricultural estate near Modriča (today a town and municipality located in Republika Srpska, an entity of Bosnia and Herzegovina). On such occasions, he also visited his father Stevan, who was retired and ill and spent a lot of time on his property in northern Bosnia. In addition to the above-mentioned painter Kravchenko, he was also interested in the work of Asen Peykov, who had lived in Belgrade for a time. This, later famous Bulgarian-Italian sculptor, made a portrait of Karamata in the shape of a bronze medallion. Karamata began devoting more and more time to business. In the mid-thirties, he and his partner, Julius Holbus, built a textile factory, which operated successfully and, before the beginning of World War II, procured the latest equipment. There is an anecdote that best describes the kind of people they were and the times they lived in. Karamata had come to Ruma to meet with a wealthy proprietor, who handed him a large sum of money to buy new machines. After the conversation, he put the money in his pocket, said his goodbyes and returned to Zemun. When asked by his wife, "Do you have a receipt for the money?" The proprietor answered in astonishment, "Why would that even cross your mind? You don't ask for a receipt from Karamata!” Just before the start of the war, Karamata became interested in objects made of wrought copper and then wrought iron. He did most of the designs himself, and set up a workshop in the backyard of his house for the blacksmiths. The smaller items, such as candlesticks or fireplace accessories, were sold through smaller porcelain and crystal shops, while the larger items, such as gates, railings or chandeliers, were made to order.

War and the New Order (1941-1950)

With the German attack on Yugoslavia, this happy and productive time, during which everything seemed to go his way, disappeared forever. Refusing to accept the Declaration to the Serbian people by the prime minister of the government under occupation, General Milan Nedić, Karamata lost his job at the University. At the end of 1941, he was offered a chair by the German Ministry of Education at a university of his choice in Germany, which Karamata did not accept.

The war was now raging all over the world and the primary issue was the issue of survival. There was no opportunity for any serious scientific work. Karamata found himself in occupied Zemun, in the Independent State of Croatia, and his colleagues, equally preoccupied with the struggle for survival, remained in Belgrade. Correspondence and contacts with colleagues abroad ceased, and modern scientific journals were no longer accessible. These were hard times, filled with injustice, human misery and tribulation. Karamata’s life was at risk. However, thanks largely to his factory, which had been profitable before the war and provided him with an enviable income, Karamata not only survived the occupation financially, but he also managed to save his life and the lives of his family. Namely, he and his partners traded silk from the factory for food. Also, when he and his wife were arrested, several times, by the Ustashas during the war – he was even on the list for Jasenovac[[11]](#endnote-11) – he saved himself primarily thanks to the corruptibility of the occupation authorities. During this whole time, according to Karamata, who took over the bookkeeping of the factory, both he and his partners obtained the necessary funds by "stealing" from themselves, forging books and presenting them as such to the occupation authorities. The factory never worked for the occupation army, although it was requested to do so. For the duration of the occupation, the blacksmith shop also worked and was somewhat profitable. However, now, in addition to the blacksmiths, all those who were enemies of the occupiers also worked here, thus finding temporary refuge. Days and years passed in uncertainty. Karamata, who was not interested in the media, either before or after the war, much less in politics, regularly listened to the BBC, despite the fact that it was strictly prohibited by the occupiers. From the outset of the bombing of Belgrade by the allies, in the spring of 1944, until the entry of Russian troops, his house was occasionally inhabited by numerous close and distant friends and acquaintances in the belief that bombs would not fall on that house and, for a while, it was called "Good People’s Hotel".

When the Partisans came to Zemun and established a new order, Karamata was again arrested due to a misunderstanding and for being wrongly categorized as a collaborator to the occupiers. His life hung in the balance. At that time, many of those who were in prison with him were summarily executed. He was rescued by "workers" who escaped death during the war by hiding in his blacksmith shop. Karamata was brought before the "People's Court" in Zemun, and since none of the summoned witnesses, the factory workers, had anything against him, he was convicted as a "war profiteer" who allegedly "ruthlessly exploited workers in his underground factories!" The factory was taken away from him and nationalized, and the blacksmith shop, which had tools and raw materials at its disposal, was reduced to only one blacksmith who, as an admirer of wrought iron, often worked there in his free time, using Karamata's designs.

With the war and the occupation, one chapter of Karamata's life ended, which included his scientific and creative work, the most striking and significant at that. World War II, as well as the period right after it, represents the most difficult period of his life and scientific work. Now, under new conditions, which he had difficulty adjusting to, the only thing left for him to do, in order to provide for himself and his family, was to go back to mathematics, his first love, with which he had lost almost all contact for more than half a decade. Correspondence with colleagues had withered away. The focus of creative scientific work had shifted to the United States. Science had continued its course, but contributions to the progress and development of new fields of mathematics were not only inaccessible, but there were many much bigger problems to deal with in times of war. Karamata, who was one of the leading mathematicians in his field of work before the war, was all the more aware of the effort needed to survive in these elite circles. Years away from the world mathematical scene, somewhat discouraged and disoriented, he nevertheless accepted the challenge. As he had already sensed, in the years prior to the war, that the golden age of his work was coming to an end, he once again, as he did then, turned to his students for inspiration and encouragement and began working with doctorate candidates. The letter he sent on October 10, 1951 from Geneva to Mitra Mitrović Đilas, the post-war Minister of Education of Serbia, testifies to his pre-war work with young people both in Belgrade (Petar Muzen and Vojislav Avakumović) and in Tübingen (Mayer-König). In presenting his idea that he could contribute the most to the University of Belgrade by rearing future staff, he writes: "I would like to note that almost all our younger doctors of mathematics, even in the days of Prof. Petrović, prepared their dissertations exclusively with me and that, before the war, I also mentored some dissertations abroad, from Belgrade." In addition to teaching and regardless of the difficulties caused by the new working conditions and the ever-increasing administrative obligations, he returned to science, devoting most of his time to doctoral students.

Finally, as soon as things began to normalize, Karamata, as a world-renowned mathematician, was returned to the position of associate professor at the Department of Theoretical Mathematics of the Faculty of Philosophy, where he assumed his lectures. And, as an already experienced professor, he once again held all his lectures in one day. He prepared them in detail, theorems and precise proofs were lined up on the board, and radiant faces came out of his classes. A series of completely different courses - elementary algebra, higher algebra, an introduction to analysis, theory of series and row theory, and descriptive geometry - lined up one after the other. In between classes, his office was full of students who brought assignments and asked for advice on their seminar papers. He also posed many mathematical problems at his lectures. Some were so difficult that at first the students didn't even understand them. He often repeated entire lectures, wanting to point out the significance of certain concepts and approaches, while some students dedicated all their work to interpreting those lectures and solving the set problems. It was not until later that the results of such hard work emerged.

The first step in writing seminars was getting acquainted with foreign literature, without which such papers could not even be imagined. If we keep in mind that the knowledge of mathematics and foreign languages ​​that came with high school was not always at the required level, we can conclude just how much effort such studies required. And with this, one can understand the disappointment of some of the young people. Because, in the end, Karamata shattered the illusions of many who came to study mathematics, who were thought to be talented mathematicians. They saw that, in addition to the talent, hard work was far more important. Students who appreciated and were able to follow his method of teaching and working, but also had the intellectual ability to comprehend the problems that Professor Karamata posed to them, greatly contributed to the continuation of his scientific work.

Although he now invested most of his effort in working with doctoral students, the work in which he found great satisfaction was very selective. He took on only those most talented and persistent as doctoral students. He worked with them almost exclusively in his house in Zemun. This working atmosphere was perhaps best felt by his first post-war doctoral student, Miodrag Tomić, who wrote: ''It was a kind of joint effort. Those who were lucky enough to enter the circle of his students and to persevere in that school will remember those days for the rest of their lives. The work would start at the table in the early afternoon and end around ten in the evening. Their eyes would well up with tears from his sharp criticism. But if he noticed a grain of truth, he would be delighted; he carved together with them and rejoiced in every success as if it were his own. To those who knew him better, his friends, comrades and students, he was a man they loved because of the warm, friendly atmosphere he brought with him. His students will remember not only the scientific work, but also the hours of joy they spent with him, forgetting that he was the professor and they students. In such an atmosphere, they found it easier to look forward to the next day of sweat and hard work, and eagerly awaited an evening of relaxation and rest.'' And what kind of students they were, but also what kind of man Jovan Karamata was, is perhaps best illustrated by his short statement: “He has surpassed me”, addressed to Vojislav Avakumović. The role that these students played in the post-war creative work of Jovan Karamata can be clearly seen from the attached list of published works. Out of 50 publications, 17 were written with associates, almost exclusively students, while out of numerous pre-war contributions, only three were realized in cooperation with V. Avakumović, H. Wendelin and M. Petrović.

Not only in our country, but also abroad, Karamata created numerous mathematicians, who, as Ranko Bojanić once said learned, among other things, to distinguish the important from the irrelevant, the trivial from the non-trivial. Many of them have even contributed to the current success and topicality of his works by transferring and incorporating his results into their works in new areas of mathematics.

Karamata believed that textbooks should be written at the end of a scientific career. Despite the fact that he was quite aware that he had already done what would be recorded in the history and development of mathematics, until his untimely death, he was actively engaged in science, thinking that there was still time to write textbooks. However, due to circumstances, he accomplished this as well. Even before the war, three papers were printed, primarily thanks to Jovan Karamata, but also to the work and activities of the Association of Mathematics Students of the University of Belgrade. After the war, Karamata wrote two official university textbooks, *Kompleksan broj* (Complex Numbers) and *Teorija i praksa Stieltjesova integrala*  (Theory and Practise of the Stieltjes integral). Although almost forgotten today, they comprise all the features of his work – from originality to content. The German mathematician, Professor Erich Kamke, suggested that he publish a monograph on Stieltjes integral in German, but because he was not satisfied with its content and text, and because he believed it should be revised to the extent that it would be a new book, he no longer had the will for such a project, and gave up on it.

Between 1945 and 1950, Karamata was extremely active in the work of the newly established Mathematical Institute of the Serbian Academy of Sciences (SAS). He presented 29 of his scientific papers at regular conferences, personally or through his closest associates, since he was often absent due to his participation in scientific conferences throughout Europe. From 1951 to 1961, when the Mathematical Institute was separated from Serbian Academy of Sciences and Arts (SASA), his scientific results were presented 15 times. In addition, in 1949 he held a course entitled ''On the Concept of Regular Variation'' (with mathematicians Dragoljub Marković and Miodrag Tomić) and, at the end of 1952, a course entitled ''Orthogonal Systems and the Fourier Series''. He was nominated to be the director of the Mathematical Institute on February 12, 1949, at the meeting of the Department of Sciences of SAS. Although at the session of the SAS Board, the Committee for Scientific Research Institutes, the University and higher education schools gave their consent for Jovan Karamata to be appointed director of the Mathematical Institute, he never officially took over the duty. He was also elected to numerous administrations, boards and commissions, which were primarily related to the Mathematical Institute and SAS.

He was elected a full member of the SAS at the main annual meeting held on March 18, 1948.

Geneva (1950-1967)

Jovan Karamata ended his official teaching career in Belgrade with his last lecture in “Analysis I”, held on January 11, 1950, when he also became a full professor at the Faculty of Sciences, University of Belgrade. The best mathematician the Serbian people had at the time, left the country. The main reason he gave was the political situation, which prevented him from appointing the most talented students to assistant positions, those who, according to him, had a future in science. Gradually, due to political or economic reasons, many of his associates left Belgrade and Yugoslavia, while those who decided to stay often worked under pressure and at non-home faculties.

The development and work of the Mathematics Institute in the post-war period did not always progress without difficulty and as easy as it might seem based on archive documents and official records of meetings of the Institute and the Serbian Academy of Sciences. Shortly after it was founded, the Institute became a thorn in the side of a group of mathematicians and a few physicists from the Faculty of Natural Sciences and the Higher Technical School in Belgrade. They, as external associates and in spite of their scientific incompetence, had the desire to reign over all mathematical institutions and control the fate of all those working in the field of mathematics by taking advantage of their good standing in the Communist Party. As winners of the war and now commissars in science, they wanted a university career. Many of them charged towards the universities, despite the lack of necessary knowledge and qualifications, to acquire higher titles than they deserved and go from being students and assistants to becoming professors and even academics (the Montenegro Academy of Arts and Sciences and the Macedonian Academy of Arts and Sciences) overnight. Meanwhile, they had contributed little or nothing to science up to that point. They immediately tore down and threw away the sign that read Belgrade Faculty of Philosophy, Mathematical Seminar Mihailo Petrović. The sign was put up in 1938, after Professor Petrović went into retirement, as a great tribute to him and everything he did for Serbian mathematics. They went so far as to provide high-quality printing for textbooks of Communist authors and professors with an unstable sense of morals, who had joined the Party, while the extremely significant textbooks of Radojčić and Karamata were retyped and copied on newspaper sheets! The majority of mathematics associates and members of the Institute, the “mathematics clique” as they called them, knew full well what the newly elected colleagues thought about them, and they lived and worked under constant surveillance, scrutiny, pressure from the government and with someone always writing down every word they say or even do not say. They lived in an environment of general insecurity, spying, reporting, persecution and arrests, which emerged in Yugoslavia after the end of the war, especially in the case of the University. This was a time called “a time of characteristics”.[[12]](#endnote-12) Characteristics were written evaluations of one’s political standing and position, morals and competency, which were prepared and written by servants of the Communist regime, including both students and professors, about their “suspicious” colleagues, and they always sent a message as to whether one is suitable for the new government or not and whether this person can be of use or is an absolute enemy. Such characteristics were delivered, without a signature or initials, to the corresponding committee where they were typed out and signed and as official documents sent back to the University. The careers and fate of many were affected by these documents. People were forced to leave the University, arrested and had their food stamps taken away. Many of the characteristics were preserved in the Archive of Serbia and the Archive of the Academy of Arts and Sciences as testimony of post-war times.

For the communist regime, Karamata was a reactionary bourgeois capitalist, not friendly to the socialism, although pretending otherwise. That minimized his professional influence in the Belgrade mathematics circles and made his life frustrating so he decided to leave to Geneva. He definitely did not take the economic side into account, considering he left his home in Zemun and found himself in an unfurnished apartment in Geneva with a family of five, each of whom brought only their personal luggage. Now, he was faced with educating three children and furnishing a household ''from square one''.

The invitation to the University of Geneva was conveyed to Karamata in late 1949 by Professor Paul E. Wenger, Dean of the Faculty of Sciences at the University of Geneva. He visited Karamata in Zemun and, on behalf of the faculty, offered him to chair the mathematics department, which had been held by the recently deceased Professor Rolin Wavre. Karamata accepted the offer for several reasons: he graduated from gymnasium in Switzerland, he spoke French fluently, and his friend and colleague, mathematician Georges de Rahm, with whom he was friends in his early youth at the gymnasium in Lausanne, also taught at the University of Geneva. He began regular classes in the summer semester of 1950, during which time he gave several lectures in Lausanne and Zurich. The official appointment in Geneva, in 1951, opened up the last chapter of his life and work.

Among the mathematicians in Geneva, in addition to de Rham, he also found the young talented André Ammann and Henri Fehr, one of the founders of the journal *L'Enseignement Mathématique*, whom he had met earlier through Mihailo Petrović. Already retired at that time, Fehr dedicated his time to the journal, and he introduced Karamata into its editorial board. After Fehr's death, in 1954 he became member of the Editorial Board and in 1955 Co-Director the journal and took over the it’s publishing. He modernized it, increased the number of readers and the circulation, and started publishing outstanding, exclusive monographs. Success was inevitable. He had taken a subsidized journal and created an issue that stood on its own two feet. His influence on the form and content of this journal was significant. He was the founder of the special series Monographies de l'Ensignement Mathematique and under his idea les Oeuvres scientifiques d'Henri Lebesgue in five volumes, were published in 1973 after JK passed away.

He was very satisfied with the atmosphere at the faculty. The faculty council consisted only of full professors, of whom there were only twenty. The minutes and transcripts were reduced to a minimum and everything was mostly based on trust, which was upheld. Among the most prominent colleagues were Jean Piaget, the well-known founder of child psychology, and Ernest Stückelberg, the theoretical physicist. In addition to his old friend Rahm, Karamata enjoyed meeting astronomer Georges Tiercy and Jean Weigle, professor of experimental physics, who as early as 1950, foreseeing the limitless future of biophysics, which later developed into molecular biology, devoted himself to this field of science and after leaving Geneva, left behind a laboratory that soon became one of the top centers for molecular biology in Europe. However, Karamata was closest with Dean Wenger, and his family, perhaps due to Ms. Wenger, who was of Serbian descent.

In Switzerland, which was not affected by World War II, the post-war period was anticipated with trepidation. However, the economic boom, which occurred only a few years after the war, opened up exceptional prospects. Thanks to a solid, war-safe infrastructure and a capable workforce, enterprising Geneva residents took advantage of the newly created conditions to attract foreign companies and international organizations. This also caused a stir in science. After the war, a branch of the Battelle Institute was opened in Geneva, which developed the most modern technology for the needs of the industry. A famous Dutch mathematician, Balthazar Van der Pol, was appointed one of the directors of the International Consultative Committee on Radiocommunications (CCIR), and in 1954 CERN was founded (the European Organization for Nuclear Research). Karamata managed relatively well in these circumstances. He actively collaborated with Van der Paul, discussed the application of mathematics in modern branches of technology with Hugo Tiemann, director of Battelle, met with professors Felix Bloch and Victor Weisskopf, the first directors of CERN, as well as many physicists and visitors to the organization. Prior to Yugoslavia's withdrawal from CERN, in 1961, among the visitors were physicists from Serbia, led by academician Pavle Savić, a good friend of Karamata's, as well as physicists from the Ruđer Bošković Institute in Zagreb.

For many, Geneva had become unavoidable, and thus Karamata's major field of study came to life. Many of Karamata's pre-war colleagues accepted invitations and gave lectures. Among the first was Hermann Weyl, who had already left mathematics at that time and devoted himself to philosophy. He was followed by George Pólya, Gábor Szegö, Karl Knopp, Wilhelm Blaschke, Heinrich Behnke, Waclaw Sierpinski, Kazimierz Kuratowski, Benoît Mandelbrot, Jean Favard, Marc Zamanski, Mark Kac, Marshal Harvey Stone, Mikhail Lavrentyev, Alfred Rényi, Paul Erdős and many others. Mathematician Đuro Kurepa, otherwise known for Kurepa tree and left factorial, was one of his many colleagues from Yugoslavia who had also been a guest.

At the University of Geneva, he was in charge of lectures on differential and integral calculus and analysis, as well as rational mechanics. While he enjoyed "juggling" the former, rational mechanics gave him headaches, in the true sense of the word. The course was held in the outdated way, in accordance with the methods of Lagrange, the father of rational mechanics. Although he was not interested in the subject, he nevertheless decided to bring the course up to date by introducing torsors and matrix calculus. It seems that this required great effort on his part. He often complained of tinnitus, which miraculously disappeared in 1962, when he left the teaching of mechanics to physicists. Karamata was not satisfied with the postgraduate studies. His method did not fall on fertile ground and was misunderstood. He quickly realized that the knowledge of the, then few, Geneva students was more substantial and broader than that of the students in Belgrade, but that they also showed less interest and were not willing to make the many sacrifices needed to solve the problems he could offer them. Still, after only a few years, he managed to attract students that suited him in Geneva as well, and he guided the doctoral dissertations of Monique Vuilleumier (1965), Ronald Rafael Coifman (1965) and Horst Baumann (1966). After coming to Geneva, trips to scientific congresses and meetings became more frequent. In addition, he has given a number of lectures in the United States, France, Romania, Hungary, and the USSR. Everywhere he went, he was a welcome guest who was received with the respect he deserved. His presentations were always attracted attention.

In the meantime, Karamata did not stop working with his Belgrade students. Since some real talents occasionally emerged among the numerous Belgrade students, though with relatively inadequate knowledge but gifted with intuition and creativity in the making, he dedicated himself to them. On the one hand, up till 1958, he regularly came to Belgrade and worked intensively with young talents, most often in groups at the SAS Mathematical Institute, but also individually, in his house in Zemun and sometimes even in Hotel Majestic. Whenever he was in Yugoslavia, he gave lectures on mathematical analysis in Novi Sad, at the Faculty of Philosophy, alternating with Professor Bogoljub Stanković. And on the other, he also worked with his students in Geneva. Miodrag Tomić, Ranko Bojanić, Milenko Šteković, Bogdan Bajšanski, Mane Maravić and others came to Geneva for a short time, in transit, or for a longer internship. During those years, he guided the doctoral dissertations of Miodrag Tomić (1950), Slobodan Aljančić (1953), Ranko Bojanić (1953), as well as being a member of the evaluation committee for Vlada Vučković (1953), Bogoljub Stanković (1954), Šefkija Raljević (1954), Bogdan Bajšanski (1956) and Milenko Šteković (1956). During his visits to Belgrade, he gave a series of lectures at the Mathematical Institute of SAS, in which he dealt with torsor calculus, general summation methods of Fourier series, slowly varying functions and their applications.

  In the post-war period, Jovan Karamata's scientific work is characterized mainly by a greater diversity of mathematical fields. He still wrote papers on classical analysis – the theory of functions and the theory of series, but also on the number theory, geometry, mathematical probability, mechanics and the history of mathematics. From these lesser-known works, one can see not only the originality of Karamata's approach to various problems, but also his broad mathematical education and versatile mathematical interests. Studying the classical and already historical results of the world's greatest mathematicians, he found room for some additions, extensions, or at least a way to simplify and “spruce up” their proofs.

Jovan Karamata’s last visit to Belgrade was in spring of 1966. He came to welcome his old friend, the famous Kazimierz Kuratowski, vice-president of the Polish Academy of Sciences. In addition to Karamata, M. Tomić and S. Aljančić were also present during his visit to SASA on April 13, 1966.

At the end of the fifties, Karamata entered a new, final phase of his life. After the untimely death of his wife Emilija, and changes he did not approve of, both at the University and at the Department of Mathematics, he slowly withdrew from Geneva’s academic life. He was increasingly absent, from December 1960 to August 1961 and then from July to September 1962, 1963 and 1964, he was a visiting researcher at the Mathematical Research Center in Madison, Wisconsin, USA. The results of the research obtained during these stays were published in papers written mainly in collaboration with Ranko Bojanić and Bogdan Bajšanski, mathematicians who left Yugoslavia in the late 1950s, for reasons similar to that of Karamata, and went to Ohio State University (Columbus), as well as Monique Vuilleumier, his faithful associate from Geneva, who joined Bojanić and Bajšanski at the University of Ohio in the late 1960s. When in Belgrade, which he now visited less often, in addition to his colleagues, he regularly met with his brothers Ozren and Kosta. During his visits with Ozren, he would recall memories from his childhood in the old family house, which was now owned by Ozren's older son, academician Stevan Karamata.

Changes were also visible in his private life. Karamata was still a night owl, but now, in addition to mathematics, he devoted more and more time to anthroposophy and the Holy Scriptures. He would often spend hours in theological discussions with Henry Babel, the first pastor of the Geneva Reformed Church. From time to time, his childhood friend, mathematician Miloš Radojčić, who lived in Thonon-les-Bains, a French town near Geneva, would also join in on these conversations. Babel, an extraordinary thinker and excellent speaker, tried to find some sort of connection between spiritual phenomena, and even concepts, with physical measures, primarily various forms of energy. As being anthroposophists, for Karamata and Radojčić spiritual concepts are a matter of faith, a kind of axiom that cannot be proven as such.

Undoubtedly, Karamata sensed his health was deteriorating. As usual, he never complained, he rarely consulted a doctor, so he was not even treated. However, at the beginning of July 1967, after a faculty session, he sat down with a colleague for coffee and said, like in roulette, "rien ne va plus". He certainly did not think that the end was that close as he was again preparing for a trip to the United States where he was to give lectures at several universities. However, after finishing with the exams on Sunday, July 12, he began bleeding heavily. The day after he was admitted to the hospital, he fell into a coma from which he never recovered. He died on August 14, 1967. Both he and his wife were heavy smokers. Cancer had taken their lives prematurely.

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Jovan Karamata lived in turbulent times. Due to the circumstances, from the time of his youth, he changed environments, both cultural and linguistic. He absorbed impressions and developed broad horizons, learned to respect the unfamiliar and foreign, developed a curiosity for the unknown and a love for travel, which remained with him for the rest of his life. In his early youth, far from his family, forced to manage completely on his own, he realized that honest work was necessary for survival. He developed an enviable power of concentration, due to which later in life he was able to disconnect from events and overcome many difficult situations. He chose a science that is largely based on logical thinking. However, he considered logic to be the lowest level of thinking, giving far more importance to intuition. He used to say that whenever he intuitively came to a desired solution, he would have the impression that the objects around him shone intensely and that the image was permanently etched in his brain. The logical part of the job – the calculation, formulation – he did more as a sculptor or painter. He had a highly developed sense of beauty. He would carve his works, rewrite them a dozen times, convinced that correct proof must also be beautiful. A much younger colleague regularly used Karamata's offprints for seminars. When asked what these old works were for, he answered "so that students can see what elegance is in mathematics". Karamata's interest in business was probably also influenced by Landau, who took a liking to him and with whom he developed a friendship. Landau was wealthy and thought that science was only for the rich, which was probably true at the time. He believed that the income, especially in the case of younger scientists, was not enough to provide for a family. Undoubtedly, Karamata had far more intuition for mathematics than for people. With regard to them, he sometimes made mistakes by accepting their faulty opinions. However, in most cases, he approached people without prejudice. He was, in the true sense of the word, a good man, always willing to help, without expecting anything in return. On some occasions, his kindness bordered on naivety so that his spontaneous interest in problems and his willingness to work were sometimes exploited. In his private life, he knew how to cheer himself up, while always remaining modest and discreet. Spiritual, turning more and more to anthroposophy and Christianity as he got older, he was not afraid of death. After a rich, fulfilling life, on his deathbed, his calm face reflected the expression of a man who had completed his mission and was now leaving satisfied. His apartment was left in order because, like in his works, he discarded everything that was unnecessary. "Elegance" was always a part of his life, as well as mathematics. In the closet, only one suit hung on a hanger. He left it there for a trip to the "new world", only to, unexpectedly, wear it to the other world. A half of a century after the death of Karamata and 90 years after the appearance of his first paper, his works remain not only like a memorial, but they are the foundation for further developments of mathematics. After his works appeared, even if they were not immediately of big importance, their value, their real value, grows with time. They survive and they don't get embedded in other results generalizing them. Other people start with them, and they don’t end with them. He created but he did not fabricate results, he invented, and he didn’t only write. And Karamata succeeded in what every scientist at the end of his career and his life would like to achieve: the work outlived its creator. However, for the history of Serbian mathematics, the well-known Karamata school of mathematics and its students, later prominent associate mathematicians, are equally as important. It seems that even today, more than 50 years since the death of Jovan Karamata, and almost 70 years since leaving the country and going to Geneva, his work and his students put a unique stamp on our topmost mathematics and confirm that, by influencing its development, Karamata has surpassed his teacher Mihailo Petrović.

The most complete survey of Karamata's theory of regularly varying functions, as well as to all its subsequent work and consequences, can be found in the mentioned monography by N. H. Bingham, C. M. Goldie and J. L. Teugels, where two chapters are dedicated to Karamata's theory. This marvellous book represents the everlasting testimony of the significance of the work of the mathematician Jovan Karamata. In spite of that, and all Karamata's mathematical contributions, neither Encyclopaedic dictionary of mathematics nor Encyclopaedia of mathematics do not mention the name of Jovan Karamata. In *Development of mathematics 1900-1950*, Basel (Birkhäuser) (1994), edited by mathematician and historian of mathematics Jean-Paul Pier, in Guidelines for the year 1930 only indicate: "J. Karamata. Tauberian theorem; slowly varying functions."

1. NOTES

   1. About the development of mathematics institutes in Serbia see Nikolić, A. (2017). [↑](#endnote-ref-1)
2. 2. Tomić, M. (1997), p. 5. [↑](#endnote-ref-2)
3. 3. Isaac Jacob Schönberg, the famous Romanian-born American mathematician, came to the same results in his doctor’s dissertation from June 1926, which he published in 1928 in the journal *Mathematische Zeitschrift* (t. 28), as did young Karamata in his dissertation. [↑](#endnote-ref-3)
4. 4. The one who failed him was the father of Slobodan Aljančić, who later became member of the Serbian Academy of Sciences and Arts and a respected mathematician, and who wrote his doctor’s dissertation under the guidance of Jovan Karamata. [↑](#endnote-ref-4)
5. 4. Tomić, M. (1997), p. 9. [↑](#endnote-ref-5)
6. 6. Journal für die reine und angewandte Mathematik 164, 1931, pp. 27-39. [↑](#endnote-ref-6)
7. 7. E. Seneta, Regularly varying functions, Lecture notes in Mathematics, 508, Springer Verlag, 1976, p. 59. [↑](#endnote-ref-7)
8. 8. The letter from 21st of May, 1969, sent to the author. [↑](#endnote-ref-8)
9. 9. Nikolić A. (2018). [↑](#endnote-ref-9)
10. 10. This is confirmed in an assumption, which the Math Student Association sent in 1935 to the dean of the Faculty of Philosophy, which was rejected as unfounded. [↑](#endnote-ref-10)
11. 11. The infamous concentration camp operated by the Croats in which thousands Serbs, Jews, the Romani and communists of various nationalities died. [↑](#endnote-ref-11)
12. 12. Trifunović D. (1995), pp. 106-132.

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