

INDICATIONS OF POSSIBLE CHAOS IN ARRAYS OF SINGLE-DOMAIN NANOMAGNETS

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ABSTRACT

We study the dynamical behaviour of a system that consists of one or two elongated nanomagnets. The magnets are coupled antiferromagnetically and subjected to periodically changing external magnetic field. The numerical simulation of the system reveals the possibility of chaotic behaviour.

THE STUDIED SYSTEM

It is well known that in continuous dynamical systems whose phase space has two dimensions, chaotic behaviour can not arise (Poincaré–Bendixson theorem). However, if there is a periodic driving force, we must add one to the number of dimensions and thus, in case of nonlinearity, chaotic motion becomes possible. In the last few years, significant attention has been concentrated on understanding the physical properties of magnetic nanosized particles [1, 2]. Both theoretical [3] and experimental [4, 5] studies indicate that the internal magnetic structure of sufficiently small particles can be regarded as a ferromagnetic monodomain. Therefore we identify each particle with one magnetic moment \vec{m}_i , where $i=1, \dots, N$, and N is the total number of particles ($N=1-3$ in this work). We use the following Hamiltonian:

$$E = -J \sum_{i=1}^{N-1} \vec{m}_i \cdot \vec{m}_{i+1} + \frac{a}{2} \sum_{i=1}^N (m_y)^2 + \frac{b}{2} \sum_{i=1}^N (m_z)^2 - \sum_{i=1}^N \vec{H} \cdot \vec{m}_i$$

where J is the usual exchange-energy term, whilst $a=10$ and $b=100$ are the anisotropy coefficients originating from shape-anisotropy. As a and b are positive, there is an effective easy x-axis and hard z-axis anisotropy at the same time. H is the applied external magnetic field, which is homogeneous and has only an x-component in this paper. When the nanomagnets are in close proximity the exchange coupling dominates the form of the coupling, coming from the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction, which can be ferromagnetic ($J>0$) or antiferromagnetic ($J<0$). We note that there is no explicit dipole-dipole coupling in this Hamiltonian. One can consider it included into the exchange-coupling term which therefore can be regarded as an effective coupling.

In order to derive the equations of motion we use the well-known Landau-Lifshitz-Gilbert equations [6], in which the first term on the right hand side causes only precession around the energetically favoured axis while the second term is the dissipation:

$$\frac{\partial \vec{m}_i}{\partial t} = -\gamma \vec{m}_i \times \vec{H}_{\text{eff}} - g \vec{m}_i \times \vec{m}_i \times \vec{H}_{\text{eff}}$$

where γ and g are constants with values $\gamma=1$ in this work. The effective field acting on the i -th particle is

$$\vec{H}_{\text{eff}} = -\frac{dE}{d\vec{m}_i}$$

To investigate the hysteresis and the magnetization reversal we apply a periodic magnetic field:

$$H_x(t) = H \cdot \sin(2\pi \cdot f \cdot t), H_y(t) = 0, H_z(t) = 0$$

where $t = j \cdot h$, j is the loop index, $h=0.001$ is the timestep.

For the numerical solution of the dynamical differential equation-system the Runge-Kutta method has been used with adaptive step-size control, written in programming language C. This means that inside one visible numerical timestep h there are actually an indefinite number of tiny time-steps. In the code the variables are the usual angles of the spherical polar coordinate system, i.e. φ and θ are the angles between the moment and the x and z axes respectively.

To characterize different behaviours, we will examine a specific stroboscopic map. It means we draw the (M_x, M_y) point-pairs periodically with two point-pairs in every $T = 2\pi / f$ time-interval, i.e. one in every half-period of the external driving field. We use the following indicator, the so called Lyapunov exponent to indicate chaotic behaviour: At a given time (let us say $t=0$), we change φ and θ with a small random number of similar magnitude and denote it by $\tilde{\theta}$ and $\tilde{\varphi}$, then calculate the logarithm of the deviation of the modified $\tilde{\theta}$ from the original value of θ divided by the initial deviation:

$$d(t) = \ln \left| \frac{\tilde{\theta}(t) - \theta(t)}{\tilde{\theta}(0) - \theta(0)} \right|.$$

Then the first few values (i.e. at the first few numerical timesteps) of this function $d(t)$ are examined. Now the Lyapunov exponent is found by $\lambda = d(t) / t$.

THE RESULTS OF THE SIMULATIONS FOR ONE OR TWO PARTICLES

We start from the point where we stopped in our previous publications: we chose the parameters f and g in the range we had already investigated [7, 8, 9]. For low frequency ($f=0.01$) and relatively strong damping ($g=0.1$) the hysteresis loop of the particle is simple and looks like a square. This simple hysteresis loop can be seen in Fig. 1. During the very short period of the magnetization-reversals the moment may move into different directions, but this does not affect the shape of the square hysteresis loop. Trajectories with different initial conditions sooner or later converge to each other. Thus the average Lyapunov exponent is not positive. After some temporary, transient oscillations due to different initial conditions, the motion can be considered periodic and the stroboscopic map contains two points.

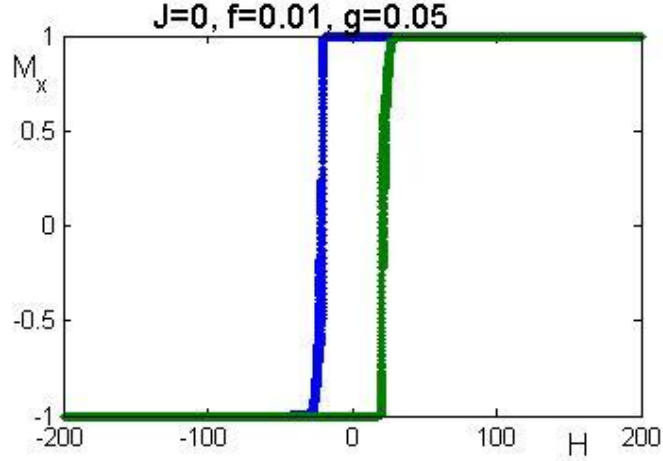


FIG.1. Square hysteresis loop. In the horizontal axis, H is the magnitude of the external field. In the vertical axis $M_x = \sum_{i=1}^N m_{i,x}$ is the x component of the total magnetization of the system, normalized by one. The data for decreasing field are denoted by blue line while for increasing field we use green line.

If the system reaches the saturation magnetisation, then the differences between different trajectories are annihilated. In order to make possible the chaotic behaviour we have to prevent this. Following the methodology employed for isolating hysteresis pathways to find the nanomechanical behaviour of nanomagnets in fluid suspensions [10], we can do it by using

- a) higher frequency
- b) weaker damping.
- c) smaller H amplitude (or additional static fields)
- d) strong antiferromagnetic coupling between the particles

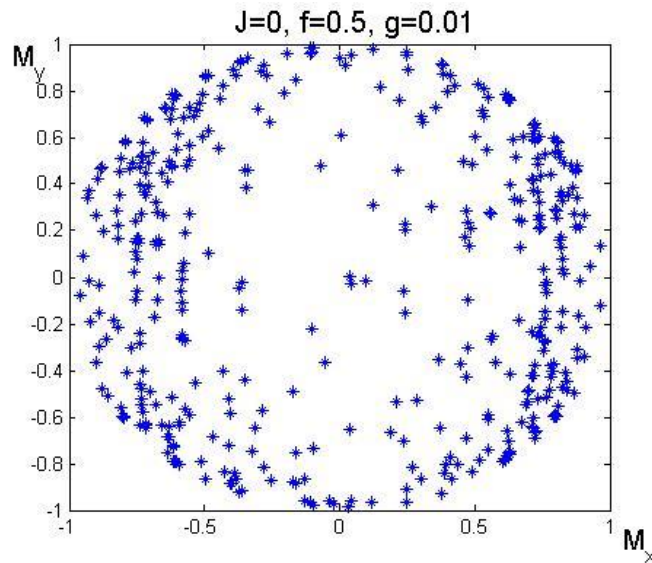


FIG.2. Stroboscopic map for one particle.

First we try a) and b) simultaneously. Higher frequency and weaker damping generally cause more irregular oscillations. The system has no time to reach the saturation and therefore the initial differences can persist between the trajectories. The stroboscopic map indicates that the motion is not periodic, see Fig. 2. However, identifying a positive Lyapunov exponent, which is a signature of an unstable orbit belonging to a chaotic region, has not occurred for the limits investigated.

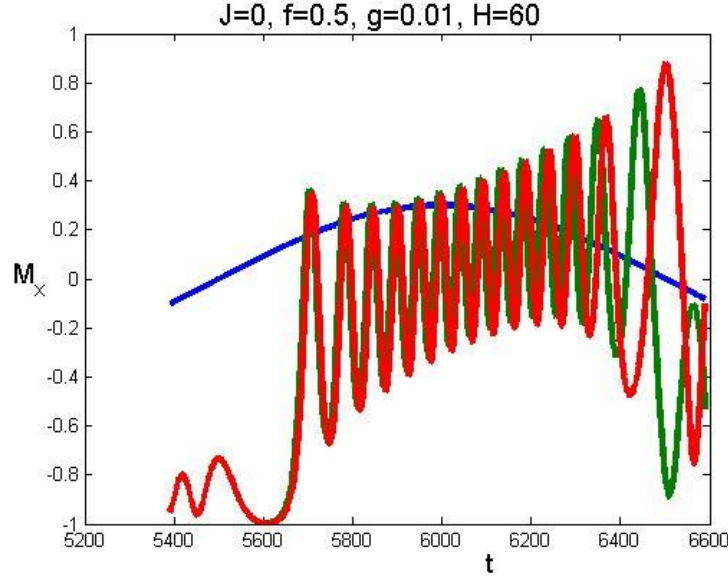


FIG.3. The red and green line are 2 trajectories initially close to each other. The blue line indicates the external field.

Applying the principles (a) to (d) of the main text leads to further system instability and a move towards a chaotic regime. The next step is to decrease H as well. At $H=60$ the field is still strong enough to force the moments to change their position between the potential wells. The initially close trajectories can move very far from each other, see Fig. 3. However, the $d(t)$ function is increasing only very slowly (see Fig. 4.), thus we can say that the behaviour is only very weakly chaotic.

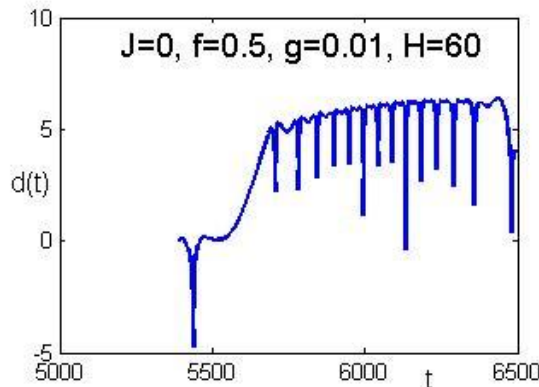


FIG.4. The $d(t)$ function to get the Lyapunov exponent $\lambda \approx 0.015$

If we set $N=2$ and $J=-20$, the behaviour becomes even more irregular. The $d(t)$ function is increasing more rapidly (Fig. 5.) and the hysteresis loops are completely distorted and random-like (Fig. 6.).

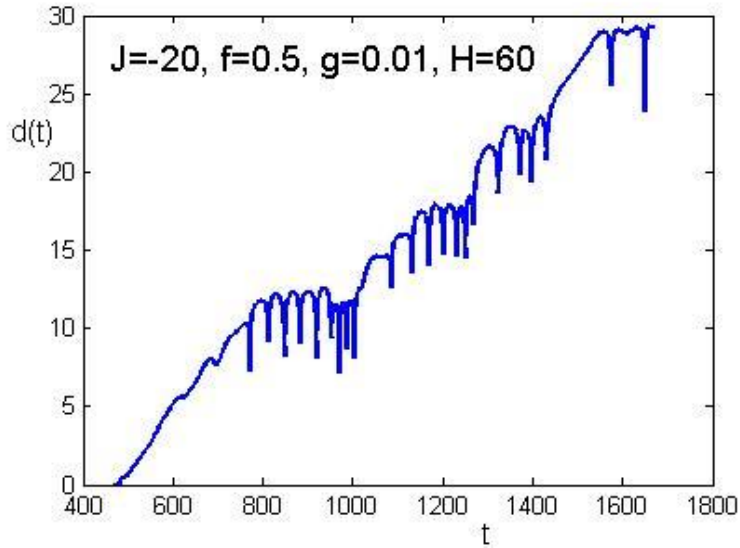


FIG.5. The $d(t)$ function for antiferromagnetic coupling, $\lambda \approx 0.027$

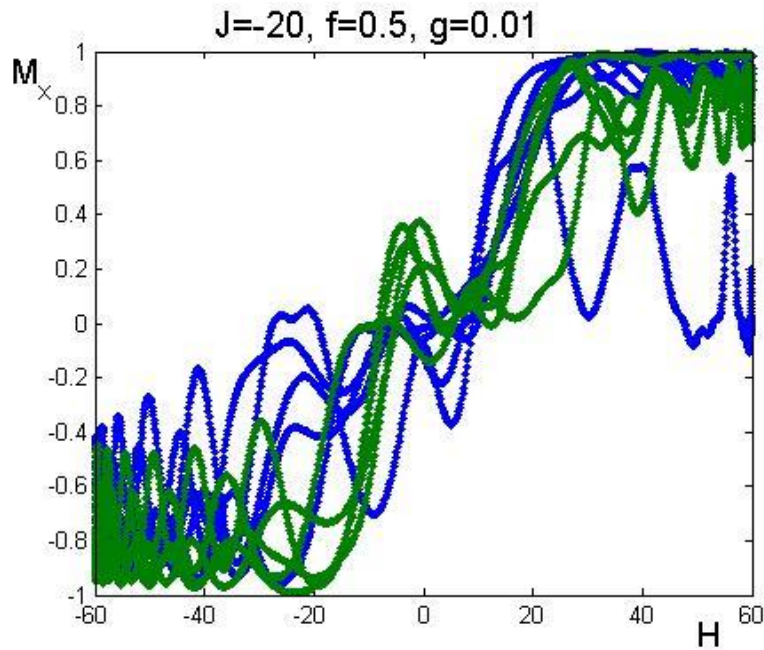


FIG.6. Hysteresis “loops” for two antiferromagnetic coupled nanomagnets.

SUMMARY

We have investigated the complex dynamical behaviour of magnetic element systems by solving the Landau-Lifshitz-Gilbert equations numerically. Even for one or two magnetic elements there is an apparently non-trivial behaviour. Future work will demonstrate larger magnetic systems too. We have previously shown that even for two magnetic elements the system can synchronise, leading to interesting

connotations for magnetic enhancement effects [8], magnetic memory devices [11] and cellular automata [12]. The chaotic regimes of nanomagnetic systems are of high interest too in order to define the operational limits of devices. The damping on the system is fundamental to the physically interesting limits of the system, but there is also a high dependence upon other parameters like shape anisotropy, the field amplitude and frequency. We find that in these magnetic systems switching occurs in accordance with the directions of the macrospins and that the attractor is always part of a limit cycle. By inhibiting the system to operate in the regime prior to reaching the saturation fields we force it to search for additional metastable states and coerce it closer to chaos. Many unusual hysteresis pathways can result as a consequence of this strategy, even when the damping is relatively high, as in magnetic memory elements. Thus, it is important to know the attractors of the system in order to maximise magnetic switching efficiency between different states. The insights gleamed from identifying the chaotic motion in this ongoing work may lead to the development of ultrafast nanomagnets and allow us to optimise the structure for spintronics in both terms of size/ geometry, as well as material choice.

ACKNOWLEDGEMENTS

For E.K. this research was (partially) carried out in the framework of the Center of Excellence of Mechatronics and Logistics at the University of Miskolc. D.M.F. thanks the EPSRC for funding under KTA grant “Developing prototypes and a commercial strategy for nanoblade technology.

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