CELL FORMATION ALGORITHMS BASED ON FORMAL CONCEPTS

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ABSTRACT

The machine-part cell formation problem is known from group technology and it aims to separate machines and parts into groups in order to obtain efficient manufacturing cells. The problem is often represented by a machine-part incidence matrix, which can be viewed as a formal context. Using the tools of formal concept analysis we determine special partitions of the set of machines which provide a base of forming cells.

1. INTRODUCTION

The cell formation problem (CF) is a frequently studied task of group technology which arises in the process of designing cellular manufacturing systems. CF involves the determination of associated groups of machines and parts. The identification of the part families is based on the similarities of the parts and their common processing requirements. These requirements are usually encoded in the machine-part incidence matrix which is the base of the cell formation process.

There are a large number of different methods proposed in the literature for solving the cell formation problem. Some of them are analytical approaches, while the others are heuristics or metaheuristics. In the review papers [10] and [11] an overall survey of the solution methods is presented, while [6] and [12] focus on the most recent techniques. Regarding our topic the most relevant solution method is introduced in [2], because the algorithms elaborated by the authors -similarly to us- use formal concepts in the process of forming machine cells.

The theory of formal concepts and the associated notions is discussed in formal concept analysis (FCA). FCA is a field of applied lattice theory which fundamentally deals with concept hierarchy. Every concept is a pair, containing a set of objects and a set of attributes with a special relation between these sets. The concepts form a partially ordered set, especially a complete lattice. Concept lattices have several applications including knowledge representation, data mining, linguistics and in many other fields. For a detailed discussion of the elements of FCA we refer to [4].

2. CELL FORMATION PROBLEM

Let M be a set of machines, $P = \{p_1, p_2, \ldots, p_k\}$ be a set of different parts and P a relation beetwen the sets with the following

DOI: 10.26649/musci.2015.045

interpretation: $(m, p) \in I$ if and only if machine processes part p. The triple is often represented by the machine-part incidence matrix ,
where $a_{ij} = 1$ if $(m_i, p_j) \in I$ and a otherwise. For example, Table 1 shows a machine-part incidence matrix from [3].

Table 1
An example of a machine-part incidence matrix

A		*Parts*										
		p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}
Machines	m_1	1	1	0	0	0	1	0	0	0	0	0
	m_2	0	1	0	0	0	1	0	0	1	0	0
	m_3	1	0	1	0	0	0	1	0	0	0	1
	m_4	0	0	1	0	0	0	1	0	0	0	0
	m_5	0	0	1	1	0	0	0	0	0	0	1
	m_6	0	0	0	1	1	0	0	0	0	1	0
	m_7	0	0	0	0	1	0	0	1	0	1	0

Starting from this matrix we have to determine the groups of the parts with similar processing requirements and the groups of the machines which process the elements of the part families. Formally, if $A \subseteq M$ and $B \subseteq P$, the pair (A, B) is called a *cell* of the triple (M, P, I). A set

of cells is called a *solution* of the cell formation problem, if forms a partition of the set M of machines and forms a partition of the set of parts, that is

– and are nonempty sets for each

– and for each

– and

After rearranging the rows and columns of the incidence matrix due to a known solution of the cell formation problem the cells can be identified from the blocks of the diagonal (see Table 2).

Table 2
Solution of the cell formation problem by rearranging the incidence matrix

A		*Parts*										
		p_4	p_5	p_8	p_{10}	p_1	p_2	p_6	p_9	p_3	p_7	p_{11}
Machines	m_6	1	1	0	1	0	0	0	0	0	0	0
	m_7	0	1	1	1	0	0	0	0	0	0	0
	m_1	0	0	0	0	1	1	1	0	0	0	0
	m_2	0	0	0	0	0	1	1	1	0	0	0
	m_3	0	0	0	0	1	0	0	0	1	1	1
	m_4	0	0	0	0	0	0	0	0	1	1	0
	m_5	1	0	0	0	0	0	0	0	1	0	1

In general, the selection of the best solution from the set of admissible solutions is a subjective decision of an expert. Aiming this decision, a measure of solution quality must be selected. In the literature there are numerous indicators for evaluating the different solutions of the problem. The common characteristics of the performance measures that they indicate efficient cells when machine utilization is of high level and the inter-cell movement of the parts is minimal. In other words good solutions contain a minimal number of 0's inside the diagonal blocks (*voids*) and a minimal number of 1's outside the diagonal blocks (exceptional elements or *exceptions*).

We define three of the most frequently used performance measures. Suppose that $S = \{(A_1, B_1), (A_2, B_2), \dots, (A_t, B_t)\}$ is a solution of the cell formation problem represented by (M, P, I). The *average machine utilization* of S is defined by

$$MU(S) = \frac{1}{t} \sum_{i=1}^{t} \frac{|(A_i \times B_i) \cap I|}{|A_i| \cdot |B_i|}$$

the *percentage of exceptional elements* of S is defined by

$$PE(S) = \frac{|I - \bigcup_{i=1}^{t}(A_i \times B_i)|}{|M| \cdot |P|}$$

and the *grouping efficacy*, introduced in [9], is

$$GE(S) = \frac{n_t - n_e}{n_t + n_v},$$

where n_t is the total number of operations, n_v is a number of voids and n_e is the number of exceptions. In an ideal case there are no exceptional elements and neither voids, and the grouping efficacy equals 1, reaching its maximal value. The grouping efficacy of the solution presented in Table 2 is

$$GE(S) = \frac{21 - 2}{21 + 6} = 0.7037,$$

since $n_t = 21$, $n_e = 2$ and $n_v = 6$.

3. SOLUTION OF THE CELL FORMATION PROBLEM USING FORMAL CONCEPT ANALYSIS

In this section we introduce some elementary notions of FCA, and show how they are applied in formulating and solving the cell formation problem. The main subjects of FCA are the formal concepts, which are defined by pairs consisting of a set of objects and a set of attributes with a strong connection between these sets. Formal concepts are originated from formal contexts. By definition, a *formal*

context is a triple $K = (M, P, I)$, where M is a set of given objects, P is a set of attributes, and $I \subseteq M \times P$ be a relation between these two sets, where $(m, p) \in$ I means that the object m has the attribute p. A formal context is often represented by a cross table: the cell in row i and column j contains a symbol x if the i-th object has the j-th attribute. As it is suggested with the same notation, a cell formation problem defined in Section 2 can be interpreted as a formal context: machines correspond to objects, parts correspond to attributes and from the machine-part incidence matrix the cross table of the context can be derived by putting an x into the cell in row i and column j if $a_{ij} = 1$.
If A $\subseteq M$, we denote by A' the common attributes of the objects belonging to A:

$$A' = \{p \in P \mid (m, p) \in I, \forall m \in A\},$$

and similarly, if $B \subseteq P$ we denote by B' the set of the objects which possesses all properties of B:

$$B' = \{m \in M \mid (m, p) \in I, \forall p \in B\}.$$

The pair (A, B) is called a *concept* of the context (M, P, I), if $A \subseteq M$, $B \subseteq P$, $A' = B$ and $B' = A$ hold. The set A is called the *extent* of the concept and the set B is called the *intent* of the concept. It is easy to see that the set $A \subseteq M$ is the extent of some concept if and only if $A'' = A$.

If (A, B) is a concept of the context (M, P, I) then $A \times B \subseteq I$, which means that the rectangle determined by the extent A and the intent B in the context is fully filled by x-s. Considering the context defined by a given cell formation problem, a concept (A, B) of the context can be identified as a cell with full machine utilization, because the machines of A process all parts belonging to B, and there is no other machine with this property. In an ideal situation the grouping efficacy of the solution of the cell formation problem equals 1, which means that each cell has maximal machine utilization and there are no exceptional elements. In this case the cells of the best solution can be selected from the concepts of the context, taking into consideration that the extents must form a partition of M, and the intents must be a partition of P.

In a general case the solutions of the cell formation problem rarely contain cells with full machine utilization (see Table 2), so we have to weaken the notion of the formal concept in order to determine the suitable partitions of the object and attribute sets. In [2], Belohlavek et al. suggested some algorithms for the cell formation process based on formal concepts. In their method the concepts of the context form kernels of possible cells, which are systematically completed by objects and attributes and this process is controlled by special functions which measure the quality of the obtained cells. In the next section a similar approach of the problem is introduced using some special partitions of the object set.

4. THE EXTENT PARTITION METHOD AND ITS MODIFICATION

Let M be an arbitrary, nonempty set and $\Pi = \{A_t \mid t \in T\}$ is a partition of M, that is A_t are nonempty, pair wise disjoint subsets of M, whose union is equal to M. The sets A_t are called the *blocks* of the partition. We define an ordering relation between the partitions of a given set: let Π_1 and Π_2 be partitions of M, then Π_1 is called *finer* than Π_2 $(\Pi_1 \leq \Pi_2)$, if every block of Π_2 is a join of some blocks belonging to Π_1. With this order the partitions of M form a complete lattice, in which the infimum of the partitions Π_1 and Π_2 is given by

$$\Pi_1 \wedge \Pi_2 = \{C \cap D \mid C \in \Pi_1, D \in \Pi_2\}.$$

If the set M is the object set of a context (M, P, I) then it has a special type of partition. A partition $\Pi = \{A_t \mid t = 1,2, \ldots, l\}$ of M is called an *extent partition* of the context (M, P, I), if every block A_t of Π is an extent of some concept originating from (M, P, I), i.e. $A_t = A_t''$ for each $t = 1,2, \ldots, l$. The blocks of an extent partitions are called *box extents*.

In [5] it has been proved that the extent partitions of a given context form a complete lattice. As a consequence of this result, for every context (M, P, I) there always exists the finest extent partition Π_0. The blocks of Π_0 are called *atomic extents* of the context. In [8] an algorithm was presented for determining all atomic extents. Based on the special characteristics of the lattice of the extent partitions the remaining box extents can be also generated by using the atomic extents (for details see [7]).

Suppose that a given cell formation problem is given by (M, P, I). We consider (M, P, I) as a context and determine its extent partitions. The blocks of an extent partitions can be viewed as the groups of machines in a possible solution of the CF problem. The following algorithm corresponds groups of parts to the blocks of a given partition in a way that the grouping efficacy of the solution should be maximal.

Algorithm 1. (input: an extent partition $\Pi = \{A_j \mid j = 1,2, \ldots, l\}$ of (M, P, I); output: part families $\{B_j \mid j = 1,2, \ldots, l\}$, for which the solution $\{(A_j, B_j) \mid j = 1,2, \ldots, l\}$ has the largest grouping efficacy available for Π.)

1. $for\ each\ j = 1,2, \ldots, l{:}\ B_j = A'_j$
2. $for\ each\ p \in P \setminus \bigcup_{i=1}^{l} A'_i\ \ and\ k = 1,2, \ldots, l\ calculate\ the\ quotients$

$$h_k(p) = \frac{\textit{number of machines in } A_k \textit{ processing } p}{|A_k|}$$

3. $B_j = B_j \cup \{p\}\ if\ \ h_j(p) = max\{h_k(p) | k = 1,2, \ldots, l\}$
$(if\ there\ is\ more\ j\ with\ this\ property, choose\ the\ least\ one)$

After determining all extent partitions and the associated part families with Algorithm 1, we choose the solution whose grouping efficacy is maximal.

Testing this method in some sample problems identified in the literature, we found, that it performs well when the best solution of the problem has relatively high grouping efficacy; that is there are only a few voids and not too many exceptions, in other words the cells can be well separated in the incidence matrix. In some cases the method did not give acceptable solutions, because it yielded only the trivial extent partition consisting the single block M. The reason of this situation is that the object set cannot be clustered well by the common attributes of its subsets.

In order to enhance the applicability of our method we have to modify it weakening the conditions which define the partitions used in the cell formation process. First we introduce another basic partition of M instead of the finest extent partition Π_0 of the atomic extents. This partition can be obtained by a slight modification of the algorithm used for determining Π_0 as follows:

Algorithm 2.

1. $F \coloneqq \{m'' \mid m \in M\}$
2. $while\ there\ are\ X, Y \in F, X \neq Y\ with\ X \cap Y \neq \emptyset$
3. $do\ F = (F \setminus \{X, Y\}) \cup (X \cup Y)$

As a consequence of line 1 all objects of M belong to some blocks of F, because $m \in m''$ for each $m \in M$. The loop in line 2 ends if all blocks of F are pairwise disjoint sets. These conditions ensure that the subsets of F form a partition of M.

Algorithm 2 differs from the original algorithm (which is used for generating the atomic extents) only in line 3, where the latter contains the equality $F = (F \setminus \{X, Y\}) \cup \{(X \cup Y)''\}$. Using operation $(.)''$ is necessary when forming atomic extents, because this operation ensures the property $X = X''$ for each block of F. In the new version this criteria is omitted, so the obtained partition F is not necessarily an extent partition of M. The blocks of F will be called the *preatoms* of the context (M, P, I).

The partition of the preatoms usually contains blocks with few elements, and the small blocks are not suitable for forming efficient cells in the cell formation problem. Unfortunately the preatoms do not have as nice properties as the atomic extents which make them available to build special structures. However the following algorithm shows an opportunity to form a greater partition than F, based on the common attributes of the objects of the preatoms.

Algorithm 3.

1. $H = F$
2. $while\ there\ are\ X, Y \in H, X \neq Y\ with\ X' \cap Y' \neq \emptyset$
3. $do\ H = (H \setminus \{X, Y\}) \cup (X \cup Y)$

Algorithm 3 forms the unions of the preatoms which have common attributes in order to obtain a partition H of M having larger blocks than F. If it is needed, the criteria in line 2 could be strengthened, permitting the union of the preatoms X and Y only if they have more than one common attributes.

As an illustration we show the detailed solution process of the problem presented in Table1. Algorithm 2 provides the partition of the preatoms as follows: $F = (\{m_1\}, \{m_2\}, \{m_3, m_4\}\ \{m_5\}, \{m_6\}, \{m_7\})$. Applying Algorithm 3 the preatoms with common attributes are unified and the result is the following partition of M: $H = \left(\{m_1, m_2\}, \{m_3, m_4, m_5\}, \{m_6, m_7\}\right)$. Finally we determined the corresponding part families to the blocks of H using Algorithm 1. The obtained solution contains the cells

$$(\{m_1, m_2\}, \{p_1, p_2, p_6, p_9\}),$$
$$(\{m_3, m_4, m_5\}, \{p_3, p_7, p_{11}\}),$$
$$(\{m_6, m_7\}, \{p_4, p_5, p_8, p_{10}\})$$

as it is depicted in Table 2.

For further testing of our method other sample problems were discussed from [1], where a two-phase method was proposed for solving the CF problem. In the first phase the groups of machines are determined by applying factor analysis and in the second phase an integer-programming method is used for identification of the part families. The authors compared their approach with other cell formation methods and proved that it performs very well in terms of a number of objective criteria. Our algorithms were tested with four incidence matrices given in the paper (with sizes $5 \times 7, 12 \times 10, 15 \times 10, 8 \times 20$) and in all cases our method gave the same solution that the authors obtained in [1]. The result encourages us to improve our concept-based method and develop further algorithms which are suitable to manage more complex problems.

ACKNOWLEDGEMENT

This research was (partially) carried out in the framework of the Center of Excellence of Mechatronics and Logistics at the University of Miskolc.

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