DEVELOPMENT OF MATHEMATICAL SKILLS WITH COMMONLY USED COMPUTER SOFTWARE

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"Education is what remains after one has forgotten everything he learned in school." — A. Einstein

ABSTRACT

One of the goals of this article is to give an answer to the question: which mathematical skills are necessary for facing the needs of twenty-first-century education? The second goal is to show one idea for how every mathematical teacher can develop these skills with widely available software.

Key words: 21st century skills, mathematical skills and abilities, learning tools, Microsoft Excel

INTRODUCTION

Life changes very quickly so we do not know what knowledge will be needed for our children in the near future. Many scientists argued that it is better to "know how" than "know what". The Hungarian mathematician Georg Pólya, who investigates methods of solving mathematical problems and ways to come to insight in mathematics education, wrote, "Our knowledge about any subject consists of information and of know-how. If you have experience of mathematical work on any level, there will be no doubt in your mind that, in mathematics, know-how is more important than mere possession of information. What is know-how in mathematics – the ability to solve problems – not merely routine problems but problems requiring some degree of independence, judgment, originality, creativity" [14, p. viii].

Hence, new standards for what students should be able to do must replace the basic skills and knowledge expectations of the past. To meet this challenge, schools must be transformed in ways that will enable students to acquire the sophisticated thinking, flexible problem solving, and collaboration and communication skills they will need to be successful in work and life [1, p.18].

Theoretical framework

Twenty-first century skills and mathematical skills

In the second chapter of the book "Assessment and Teaching of 21st Century Skills" researchers compared twelve relevant frameworks drawn from a number of countries, and organised skills necessary for all people in the "KSAVE[1] Model of

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[1]Knowledge, Skills, Attitudes, Values and Ethics

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Twenty-First Century Skills framework". These include *Ways of Thinking* (Creativity and innovation; Critical thinking, problem solving, decision making; Metacognition), *Ways of Working* (Communication; Collaboration), *Tools for Working* (Information literacy, ICT literacy), *Living in the World* (Citizenship; Life and career; Personal and social responsibility) [1, p. 36-58]. Mathematical education is closely related to the first group but the others may and must be improved directly and indirectly, too. Creativity and innovation skills are operationally defined from Jane Piirto [13].

Defined from Mbewe [8, p. 81], we considered as mathematical skills the basic and integrated **science process skills (**Observation, Classification, Quantification, Measurement, Inferring, Communication, Formulating hypothesis, Experimenting, Making operational definition, Interpreting data, Predicting, Controlling variables, and Using space/time relations).

Since the 1970s the NCSM[2] has divided basic **mathematical skills** into ten areas. All are basic to pupils' development of the ability to reason effectively in varied situations: Problem Solving; Applying Mathematics to Everyday Situations; Alertness to the Reasonableness of Results; Estimation and Approximation; Appropriate Computational Skills; Geometry; Measurement; Reading, Interpreting, and Constructing Tables, Charts, and Graphs Using Mathematics to Predict; and Computer Literacy [11, p. 4-6].

The National Center for Education Statistics in 1995 outlined "student skills required for success in mathematics" [9, p. 212]: Remember formulas and procedures; Think in sequential manner; Understand concepts; Think creatively; Understand math use in real world; and Support solutions.

The **NCTM "process standards"** are the first standards with longstanding importance in mathematics education in the USA and Canada. They include problem solving, reasoning and proof, communication, representation, and connections [10].

Next are the strands specified in the National Research Council's report "Adding It Up" [5, p. 116]:

- conceptual understanding – comprehension of mathematical concepts, operations, and relations
- procedural fluency – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- strategic competence – ability to formulate, represent, and solve mathematical problems
- adaptive reasoning – capacity for logical thought, reflection, explanation, and justification
- productive disposition – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Binkley divides *Cognitive Process Skills* [1, p. 47] into the following categories: Task Regulation Skills (Organises (problem analysis); Set goals; Resource management; Flexibility and ambiguity; Collects elements of information; Systematicity), Learning and Knowledge Building Skills (Relationships (represents and formulates); Rules: "If ... then"; Hypothesis "what if..." (reflects and monitors)).

[2]Network Communicate Support Motivate

Thinking skills with technology

Problem solving skills in a technology-rich environment will become increasingly important [3, p. vii].

Thinking skills need to be explicitly modelled, drawn out and reapplied in different contexts. Effective teaching for transferable thinking skills with ICT contains some or all of the following elements:

- teaching a 'thinking' vocabulary and giving learners an explicit explanation of the thinking skills that they are to be learnt
- observing an expert performing the task (modelling)
- giving timely feedback on performance (formative assessment)
- direct support in the early stages of learning a task (scaffolding) and then a gradual move towards self-regulation and autonomy (teacher fade-out)
- the opportunity to articulate thinking strategies and discuss these with other learners (thinking together)
- the explicit demonstration of how thinking strategies acquired in one subject area can be used to solve problems in another area (bridging) [15, p. 34].

Collaborative problem-solving skills are considered necessary for success in today's world of work and school [1, p. 32].

Higher order thinking skills (in Bloom's learning Taxonomy) include critical, logical, reflective, metacognitive, and creative thinking. These are activated when individuals encounter unfamiliar problems, uncertainties, questions, or dilemmas. Successful applications of the skills result in explanations, decisions, performances, and products that are valid within the context of available knowledge and experience and that promote continued growth in these and other intellectual skills. Higher order thinking skills are grounded in lower order skills such as discriminations, simple application and analysis, and cognitive strategies and are linked to prior knowledge of subject matter content. Appropriate teaching strategies and learning environments facilitate their growth as do student persistence, self-monitoring, and open-minded, flexible attitudes [6, p. 35].

How to identify Mathematical skills

Borovik and Gardiner describe traits of mathematically able children [2, p. 4]:

- Ability to make and use generalisations — often quite quickly, rapid and sound memorisation of mathematical material.
- Ability to concentrate on mathematics for long periods without apparent signs of tiredness.
- Ability to offer and use multiple representations of the same mathematical object. (They switch easily between representations of the same function by tables, charts, graphs, and analytic expressions.)
- An instinctive tendency to approach a problem in different ways: even if a problem has been already solved, a child is keen to find an alternative solution.
- Ability to utilise analogies and make connections.
- Preparedness to link two (or more) elementary procedures to construct a solution to a multi-step problem.

- Ability to recognise what it means to “know for certain”.
- Ability to detect unstated assumptions in a problem, and either to explicate and utilise them, or to reject the problem as ill-defined.
- A distinctive tendency for “economy of thought,” striving to find the most economical ways to solve problems, for clarity and simplicity in a solution.
- Instinctive awareness of the presence and importance of an underlying structure.
- Lack of fear of “being lost” and having to struggle to find one’s way through the problem.
- A tendency to rapid abbreviation, compression or a curtailment of reasoning in problem solving.
- An easy grasp of encapsulation and de-encapsulation of mathematical objects and procedures.

Summary

To sum up, after analysing of different frameworks and notions as mathematical competency, mathematical literacy, mathematical proficiency we identify mathematical skills that educators must develop in their learners:

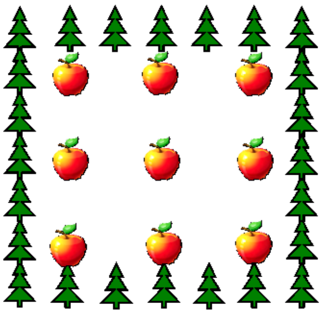
- Problem solving skills:
  - Mathematising [12, p. 51] – skill for creating mathematical models from real-life problems
  - Interpreting data, selecting and controlling variables
  - Specifying relationships
  - Choosing appropriate computation
  - Constructing viable arguments
  - Observing, classifying, using space/time relationships
  - Inferring, predicting
  - Being alert to the reasonableness of results
  - Setting goals and persevering to meet them;
- Thinking skills –being able to:
  - Reason abstractly and quantitatively
  - Find regularity in repeated reasoning
  - Convergent thinking
  - Divergent thinking (innovative, creative thinking)
  - Inductive and deductive reasoning
  - Critical thinking
  - Reflective thinking;
- Additional skills in performing mathematics:
  - Using appropriate tools strategically
  - Attending to precision
  - Procedural fluency
  - Looking for and using structures
  - Communicating mathematically
  - Collaboration skills
  - Applying mathematics to everyday situations

- Metacognitive skills.

DEVELOPING MATHEMATICAL SKILLS WITH COMMONLY USED COMPUTER SOFTWARE

We suggest to teachers to develop the mathematical skills of students in school with widespread software like Microsoft Excel for the following reasons: you do not need to search for the program and ask a supervisor to install it; every school has this software on the computers in the laboratory; students learn how to use it in classes of Information Technology; after graduating it is very likely they will use this software in their work; and Excel is a part of the Microsoft Office suite so you can transfer the knowledge and skills as objects that you made in one of the application to other applications of the Office package. Students may use simple objects, like a table to recognise a pattern:

n = 3



n	Number of apple trees	Number of fir trees
1	1	8
2	4	
3		
4		
5		

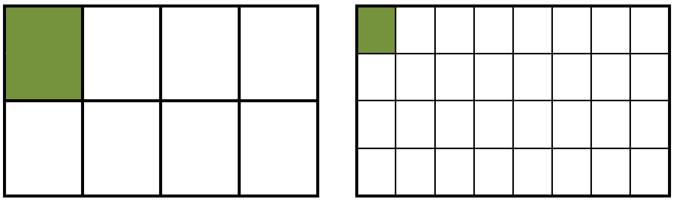
Fig.1: A problem from PISA 2012

They may write formulae, fill automatically with drag down, observe relationships and draw the graph of a function:

S	v	t
40	40	1
80	40	2
120	40	3
160	40	4

Fig. 2: Draw graph from the table

School pupils can understand deeper mathematical notions using a grid for visualisation.



$$\left(\frac{1}{2}\right)^1 > \left(\frac{1}{2}\right)^2 > \left(\frac{1}{2}\right)^3 > \left(\frac{1}{2}\right)^4$$

Fig. 3: Compare by visualising part of whole

First they can use the educational tools ready made by their teacher to gain immediate feedback after solving a part of a problem in their notebook. Next, they will be more interested in inventing their own “calculators” so they may improve their logical thinking:

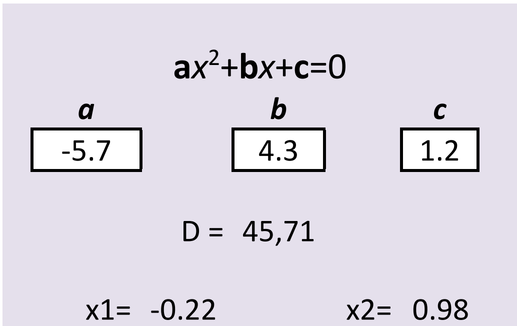


Fig. 4: Educational tool for solving a second order equation

Excel has simple drawing functions. For example learners may be challenged to draw the intersection point of perpendicular bisectors of triangle.

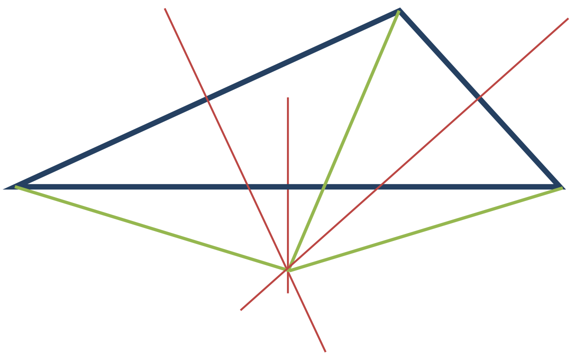


Fig. 5: Perpendicular bisectors of an obtuse-angled triangle

Electronic tables used like a table or grid, charts, graphs of functions, simple drawing and concept mapping can all be used to support critical and analytical thinking about relationships within mathematical concepts. Microsoft Excel has many other more sophisticated abilities which we do not discuss in this article. These examples are enough to show that every math teacher who knows what skills he has to develop in his students, even he did not study subjects like Information Technology previously, can make use of computers in some of his classes.

We can remark that not all mathematical classes can be conducted in computer labs and “Learners need to be given direct support in the early stages of learning a task (scaffolding) and to move gradually towards self-regulation and autonomy (teacher-fading)” [15, p. 32]. Meanwhile the use of ITC is a great advantage in enlarging the possibility of developing skills and abilities “With the digitalization of education various electronic tools (hardware and software) aid the measurements. These new digital technologies are excellent for application in the educational process (e.g. during motivation, repetition, exercise or rating)” [16, p.77.].

We plan to further develop effective educational tools and methodological structures using Microsoft Excel in mathematical education.

The motivation for the use of ITC in classes could be formulated as “Teaching mathematics has an interesting stance from the didactic point of view and the didactics of digital education is becoming one of the most basic questions. Classical and innovative educational methods and forms gain a new dimension in a digital environment, however new methods are appearing which are directly focused on using ICT.” [17, p. 198.]

CONCLUSION

Technology advances very quickly and our life is always changing, so educational goals must be revised periodically. Learners have to know which skills they must possess. Mathematics education must develop higher-order thinking skills in such a way that students can apply them not only in intra-mathematical situations but in real life, too.

“Thinking mathematically and solving mathematical problems successfully are skills which evolve in the course of learning mathematics in a school context, as well as in an everyday context” [7, p. 33]. Using everyday spreadsheet software to demonstrate and practice mathematics skills is likely to aid teachers and students in developing these skills.

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