

COMPOSITE BEAM WITH WEAK SHEAR CONNECTION SUBJECTED TO THERMAL LOAD

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Abstract

This paper gives an analytical solution for deflection, slip and internal forces in composite beams with weak shear connection. The applied loads are the mechanical and thermal load. The thermal load is caused by uniform temperature change and the considered beam is statically indeterminate. The Euler-Bernoulli beam hypothesis is assumed to hold for both two beam components. The constitutive equation between the horizontal slip and inter-laminar shear force is linear. An example illustrates the application of presented method.

1. INTRODUCTION

The paper deals with the solution of static problem of two-layer composite beam with weak shear connection. The considered beam and its load are shown in Fig. 1. The beam carries the uniform mechanical load at high temperature, so that the temperature change $T = t - t_0$, where t is the absolute temperature of the beam and t_0 is the reference temperature at which no deformation and the beam is stress free. The presented analytical solution is based on the Euler-Bernoulli beam theory and the one-dimensional version of Duhamel-Neumann's law [1,2]. The beam component B_i has a rectangular cross section A_i whose dimensions are h_i and b ($i=1,2$) as presented in Fig. 1. The modulus of elasticity for beam component B_i is E_i and the linear thermal expansion coefficient is α_i ($i=1,2$). The length of the beam is denoted by L and the cross section at $z=0$ is fixed and the cross section at $z=L$ is simply supported. The origin O of the rectangular Cartesian coordinate system $Oxyz$ is the E -weighted centre of the left end cross section, so that the axis z is the E -weighted centre line of the considered composite beam with flexible shear connection. A point P in $B = B_1 \cup B_2$ is indicated by the position vector $\overrightarrow{OP} = \mathbf{r} = x\mathbf{e}_x + y\mathbf{e}_y + z\mathbf{e}_z$, where $\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$ are the unit vectors of the coordinate system $Oxyz$. It is known the position of E -weighted cross section is obtained from next equation (Fig. 1)

$$c_1 = \left| \overrightarrow{CC_1} \right| = \frac{A_2 E_2}{\langle AE \rangle} c, \quad c_2 = \left| \overrightarrow{CC_2} \right| = \frac{A_1 E_1}{\langle AE \rangle} c, \quad \left| \overrightarrow{C_1 C_2} \right| = c = \frac{h_1 + h_2}{2}, \quad \langle AE \rangle = A_1 E_1 + A_2 E_2. \quad (1)$$

The common boundary of the beam components B_1 and B_2 is determined by $y = y_{12} = c_1 - 0.5h_1$, $|x| \leq 0.5b$, $0 \leq z \leq L$.

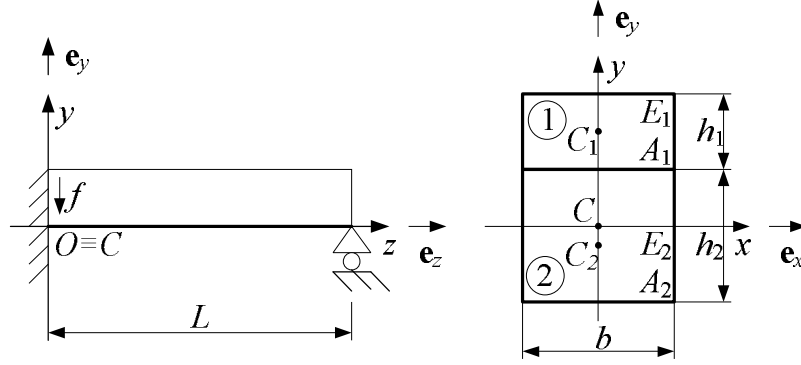


Figure 1. Two-layer composite beam

2. GOVERNING EQUATIONS

According to the Euler-Bernoulli beam theory, which is valid for each homogeneous beam components, the deformed configuration is described by the next displacement field

$$\mathbf{u}(x, y, z) = v(z)\mathbf{e}_y + \left(w_i(z) - y \frac{dv}{dz} \right) \mathbf{e}_z, \quad (x, y, z) \in B_i, \quad (i = 1, 2). \quad (2)$$

On the common boundary of B_1 and B_2 the axial displacement may have jump which is called the interlayer slip

$$s(z) = w_1(z) - w_2(z). \quad (3)$$

Application of the strain displacement relationship of the linearized theory of elasticity gives [1,2]

$$\varepsilon_x = \varepsilon_y = \gamma_{xy} = \gamma_{yz} = \gamma_{xz} = 0, \quad (x, y, z) \in B, \quad (4)$$

$$\varepsilon_z = \frac{dw_i}{dz} - y \frac{d^2v}{dz^2}, \quad (x, y, z) \in B_i, \quad (i = 1, 2), \quad (5)$$

where $\varepsilon_x, \varepsilon_y, \varepsilon_z$ are the normal strains, $\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$ denote the shearing strains. The normal stress σ_z is obtained from the one-dimensional version of Duhamel-Neumann's law [1,2]

$$\sigma_z = E_i \left(\frac{dw_i}{dz} - y \frac{d^2v}{dz^2} - \alpha_i T \right), \quad (x, y, z) \in B_i, \quad (i = 1, 2). \quad (6)$$

The temperature of the two-layer composite beam initially is the reference temperature t_0 . Its temperature is slowly raised to constant uniform temperature $t = t_0 + T$. Below we define the section forces and section moments [3] (Fig. 2)

$$N_i = \int_{A_i} \sigma_z dA = A_i E_i \left(\frac{dw_i}{dz} - (-1)^{i+1} c_i \frac{d^2 v}{dz^2} - \alpha_i T \right), \quad (i=1,2), \quad (7)$$

$$M_i = \int_{A_i} y \sigma_z dA = A_i E_i (-1)^{i+1} c_i \left(\frac{dw_i}{dz} - \alpha_i T \right) - E_i I_i \frac{d^2 v}{dz^2}, \quad (i=1,2), \quad (8)$$

where

$$I_i = \int_{A_i} y^2 dA, \quad (i=1,2). \quad (9)$$

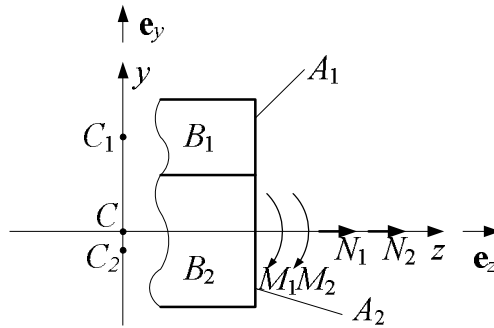


Figure 2. Normal forces and bending moments

The interlayer slip s is assumed to be a linear function of shear force transmitted between the two beam components that is we have [3,4]

$$Q = ks, \quad (10)$$

where k is the slip modulus. In the present problem there are no axial forces, so $N = N_1 + N_2 = 0$, that is

$$A_1 E_1 \frac{dw_1}{dz} + A_2 E_2 \frac{dw_2}{dz} - \langle AE \alpha \rangle T = 0, \quad \langle AE \alpha \rangle = A_1 E_1 \alpha_1 + A_2 E_2 \alpha_2. \quad (11)$$

Combination of Eq. (3) with Eq. (11) provides

$$N_1 = \langle AE \rangle_{-1} \left[\frac{ds}{dz} - c \frac{d^2 v}{dz^2} + (\alpha_2 - \alpha_1) T \right], \quad (12)$$

$$N_2 = \langle AE \rangle_{-1} \left[-\frac{ds}{dz} + c \frac{d^2 v}{dz^2} + (\alpha_1 - \alpha_2) T \right], \quad (13)$$

where

$$\frac{1}{\langle AE \rangle_{-1}} = \frac{1}{A_1 E_1} + \frac{1}{A_2 E_2}. \quad (14)$$

The bending moment on the whole cross section can be expressed as

$$M = c \langle AE \rangle_{-1} \left[\frac{ds}{dz} + (\alpha_2 - \alpha_1) T \right] - \{IE\} \frac{d^2 v}{dz^2}, \quad \{IE\} = I_1 E_1 + I_2 E_2. \quad (15)$$

The cross sectional shear force is as follows

$$V(z) = \frac{dM}{dz} = c \langle AE \rangle_{-1} \frac{d^2 s}{dz^2} - \{IE\} \frac{d^3 v}{dz^3}. \quad (16)$$

From the equilibrium condition of a small beam element ΔB_1 (Fig. 3) we receive

$$\frac{dN_1}{dz} - ks = 0. \quad (17)$$

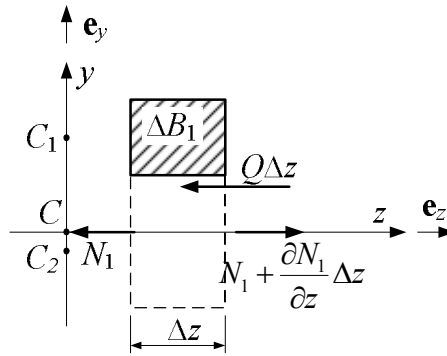


Figure 3. Equilibrium condition in axial direction

In detailed form of Eq. (17)

$$\frac{d^2 s}{dz^2} - \frac{k}{\langle AE \rangle_{-1}} s - c \frac{d^3 v}{dz^3} = 0. \quad (18)$$

Combination of Eq. (16) with Eq. (18) results

$$\frac{d^2 s}{dz^2} - \Omega^2 s + \frac{c}{\langle IE \rangle} V = 0, \quad (19)$$

where

$$\langle IE \rangle = \{IE\} - c^2 \langle AE \rangle_{-1}, \quad (20)$$

$$\Omega^2 = k \frac{\{IE\}}{\langle AE \rangle_{-1} \langle IE \rangle}. \quad (21)$$

The cross sectional rotation in terms of deflection is

$$\psi(z) = -\frac{dv}{dz} \quad (22)$$

according to the Euler-Bernoulli beam theory. From Eq. (15) we get

$$\begin{aligned} \{IE\}(\psi(z) - \psi(0)) &= M_I(z) - c \langle AE \rangle_{-1} [s(z) - s(0) + (\alpha_2 - \alpha_1)Tz], \\ M_I(z) &= \int_0^z M(\zeta) d\zeta. \end{aligned} \quad (23)$$

Integration of Eq. (23) yields the expression of $v = v(z)$

$$\begin{aligned} \{IE\}[v(z) - v(0) - \psi(0)z] &= -M_{II}(z) + c \langle AE \rangle_{-1} \left[\int_0^z s(\zeta) d\zeta - s(0)z + 0.5(\alpha_2 - \alpha_1)Tz^2 \right], \\ M_{II}(z) &= \int_0^z M_I(\zeta) d\zeta. \end{aligned} \quad (24)$$

Eqs. (20), (23) and (24) with the boundary conditions give the possibility to obtain the deflection, slip and cross sectional rotation. The application of followed method is illustrated by the solution of problem depicted in Fig. 1.

3. EXAMPLE

Denote F the unknown reaction at $z = L$. By the application of equation of statics we gain

$$V(z) = F + f(z - L), \quad (25)$$

$$M(z) = F(z - L) + \frac{f}{2}(z - L)^2. \quad (26)$$

The boundary conditions in our case are

$$v(0) = 0, \quad \psi(0) = 0, \quad s(0) = 0, \quad v(L) = 0, \quad N_1(L) = 0. \quad (27)$$

It can be proved that from Eq. (27)₅ and $M(L) = 0$ it follows that

$$\frac{ds}{dz} = (\alpha_1 - \alpha_2)T \quad \text{at } z = L. \quad (28)$$

Eqs. (24), (27)₃ and Eq. (28) can be used to get the solution of Eq. (19) in terms of F . Substitution of expression of $s = s(z)$ into Eq. (26) and using the boundary conditions (27)₁, (27)₂ and (27)₄ we get the value of the reaction F , which

essentially the solution of the considered problem since $V = V(z)$, $M = M(z)$ will be known functions. The following numerical data are used in the example: $h_1 = 0.03$ m, $h_2 = 0.06$ m, $E_1 = 1.22 \times 10^{10}$ Pa, $E_2 = 8 \times 10^{10}$ Pa, $b = 0.01$ m, $L = 1.5$ m, $k = 6 \times 10^7$ Pa, $\alpha_1 = 2.8 \times 10^{-6}$ 1/K, $\alpha_2 = 1.43 \times 10^{-5}$ 1/K, $T = 250$ K, $f = 1000$ N. The computations result for the reaction at $z = L$, $F = -255.70048$ N. In Fig. 4 the graph of deflection function and in Fig. 5 the graph of slip function are shown for $f \neq 0$, $T \neq 0$. The plots of bending moment $M = M(z)$ obtained from formula (15) and formula (26) are illustrated in Fig. 6 for $f \neq 0$, $T \neq 0$. The plot of the axial force $N_1 = N_1(z)$ is presented in Fig. 7 for $f \neq 0$, $T \neq 0$. If there is no applied thermal load then $f = 1000$ N and $T = 0$, the plot of $v = v(z)$ and $s = s(z)$ are shown in Figs. 8 and 9. In this case the bending moment $M = M(z)$ and axial forces can be seen in Figs. 10 and 11.

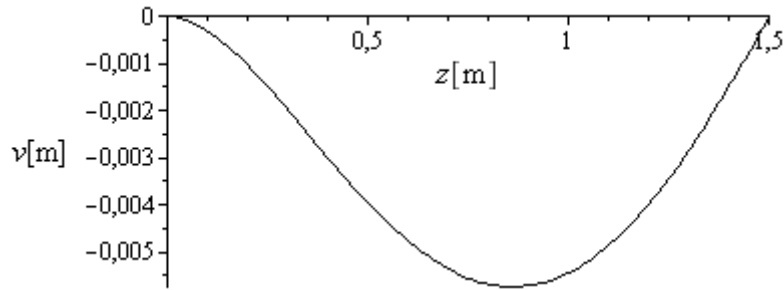


Figure 4. The graph of $v = v(z)$ ($f \neq 0, T \neq 0$)

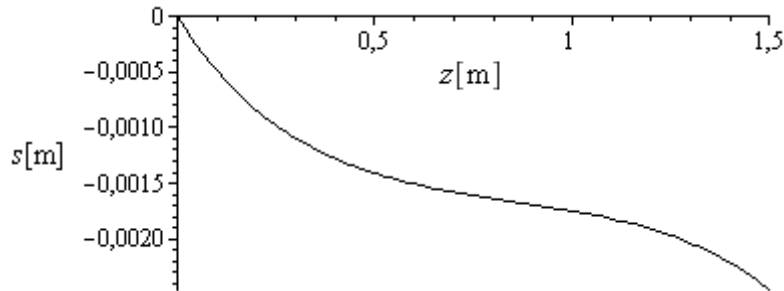


Figure 5. The graph of $s = s(z)$ ($f \neq 0, T \neq 0$)

4. CONCLUSIONS

This paper presents an analytical method to obtain the deflection, slip and internal forces for composite beams with weak shear connection subjected to mechanical and thermal load. The solution of this strength of materials problem can be used to design composite beams with imperfect connection working in high temperature.

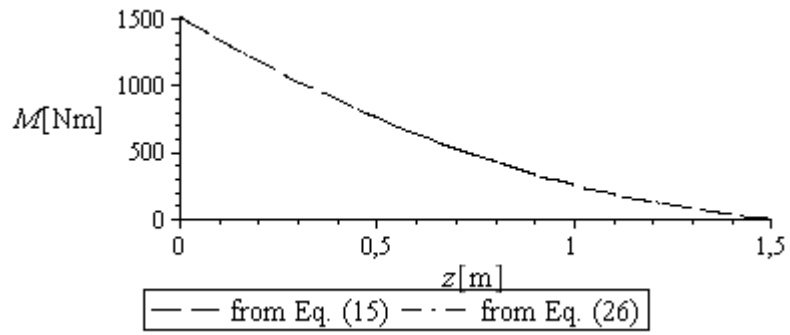


Figure 6. The plot of $M = M(z)$ ($f \neq 0, T \neq 0$)

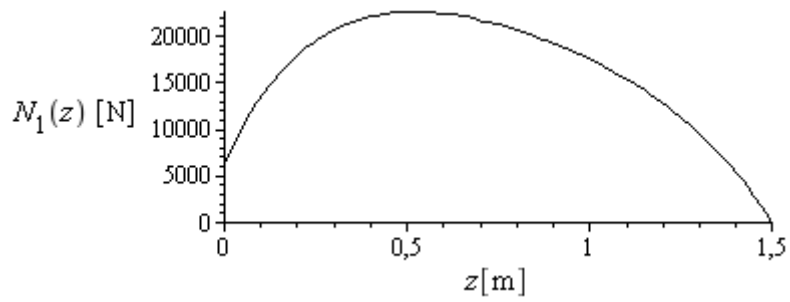


Figure 7. The plot of $N_1 = N_1(z)$ ($f \neq 0, T \neq 0$)

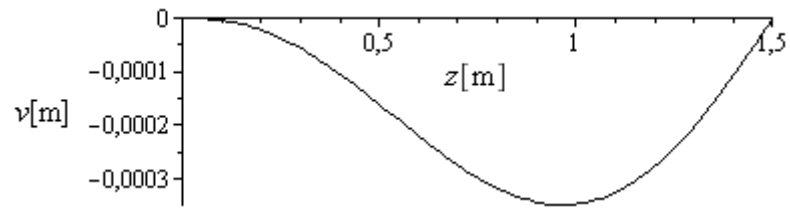


Figure 8. The plot of $v = v(z)$ ($f \neq 0, T = 0$)

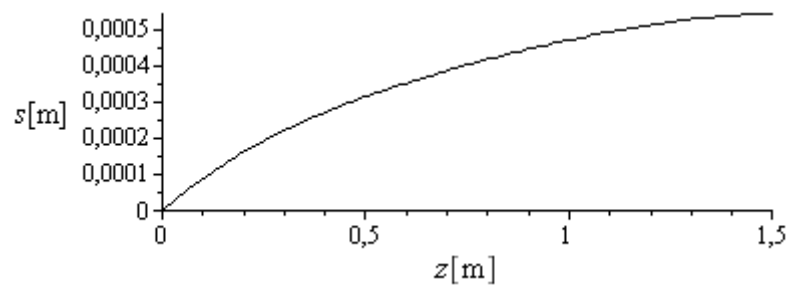


Figure 9. The plot of $s = s(z)$ ($f \neq 0, T = 0$)

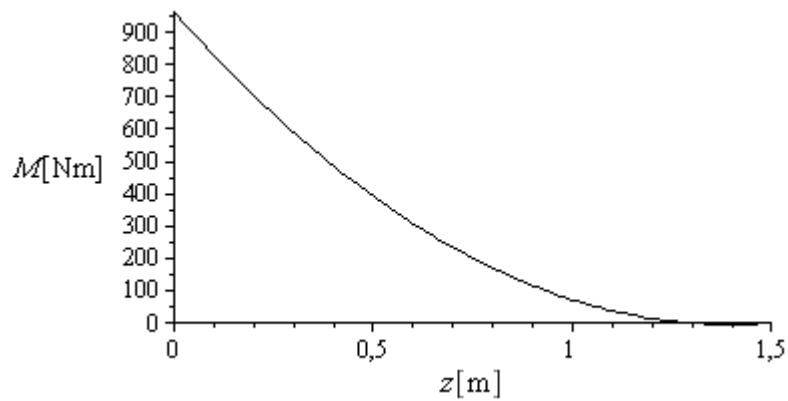


Figure 10. The graph of $M = M(z)$ ($f \neq 0, T = 0$)

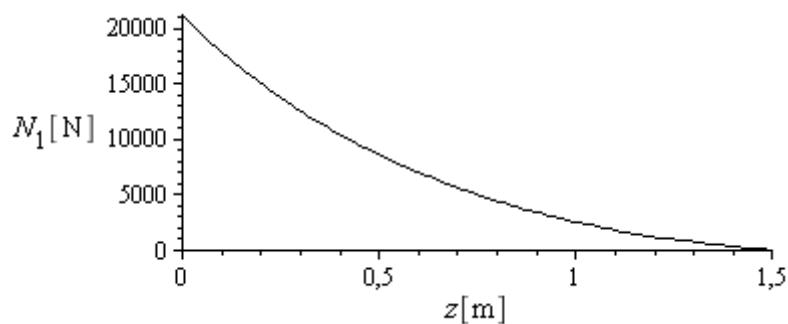


Figure 11. The graph of $N_1 = N_1(z)$ ($f \neq 0, T = 0$)

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