HIRAULIC DESIGN OF AN IMPELLER OF A CROSS-FLOW TURBINE

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ABSTRACT

This article, as a synthesis of our results described in our previous articles, deals with the formulae necessary to numerically calculate the values that are included in the basic data system of a cross-flow turbine, determined by its operation and geometry. We handled the data valid to the circumference of the impeller that are related regarding optimizing the losses along the circumference and those that are related regarding the optimization to obtain an angular momentum free exit separately. The two data systems are interdependent, therefore a complete redesigning of the data may be required to obtain a suitable compromise. We gave a method to calculate the position of the radius widening out inside the impeller, which can be used for estimating the expected extent of the impact phenomena. Finally, we indicated the formulae to calculating blading geometry up to the practical requirements.

1. INTRODUCTION

The geometry of an impeller of a cross-flow turbine must be designed in a way that the operation under the desired operating conditions can take place with the least losses possible. The blading is cylindrical and the blades have a circular arc shape (Figure 1). The initial data for the geometrical design are as follows: the rate of flow, the speed of the impeller and the available head. This article summarizes the relations, with which the geometrical design that provides the desired operating conditions and minimizing the key types of losses can be co-ordinated. The „geometrical design" process consists of harmonizing the dimensions of the impeller and the components of the velocity triangles.

Taking the speed, the rate of flow and the head into account, we calculate the proportions of the impeller so that the exit be *angular momentum free*. With an exit *not free of angular momentum*, even significant losses may occur. Other sources of loss are the *shock* from the change in the flow direction at the entry and the *exit loss* caused by the kinetic energy being lost due to the exiting mass flow. The *shock loss* can be influenced in an advantageous way by means of an appropriate geometrical design. The key is to keep the through-flow inside the impeller as *impact-free* as possible. There is always a potential of impacts at the shaft across the impeller. Im-

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pacts may also take place at the inner circumference of the blading. In order to avoid such impacts, the position of the flow jet must be known – this depend on the operating state.

This article builds on our earlier results detailed in our previous publications [1]-[7]. Therefore the calculation formulae still refer to the one-dimensional and blade-congruent flow of an incompressible medium. This allows significant simplifications, but the results still often yield complex calculation formulae. Therefore we, wherever possible, also indicate the calculation formulae in the form of diagrams. The accuracy these diagrams can be read with is limited, but this complies with the errors resulting from the approximations in the basis of the calculation model.

2. THE DATA SYSTEM AT THE CIRCUMFERENCE OF THE IMPELLER THAT DESCRIBES THE OPERATING STATE OF A CROSS-FLOW TURBINE

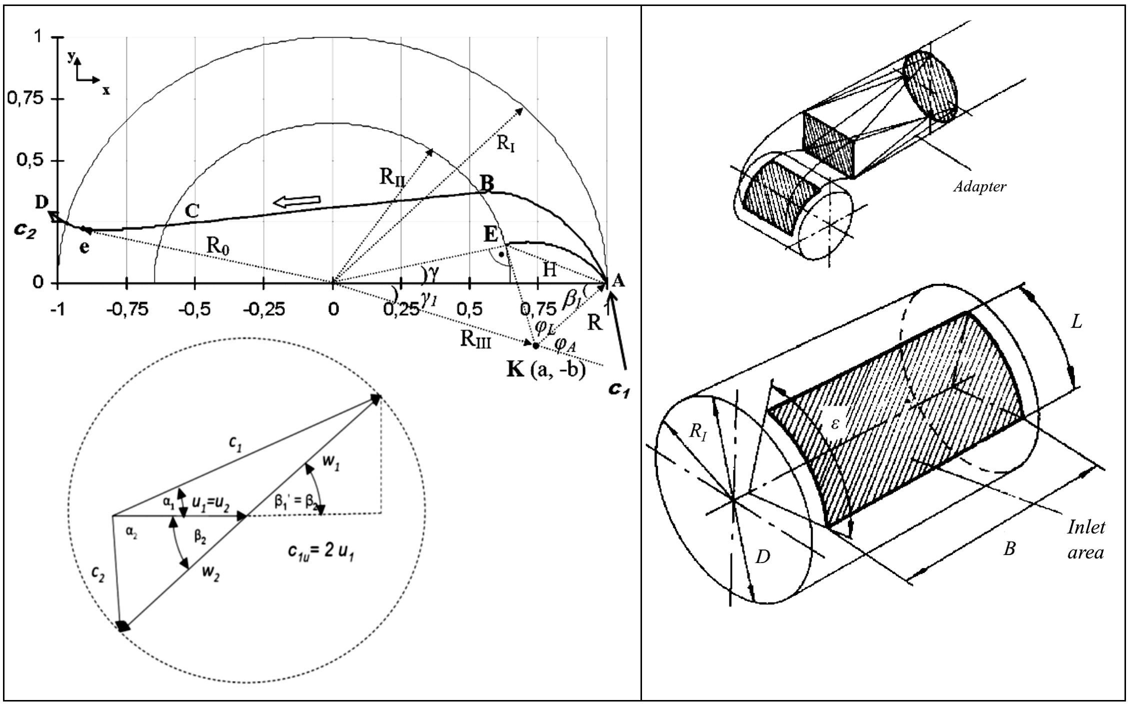

Figure 1

Figure 1 shows the velocity triangles in case of an *angular momentum free outlet* and the most common configuration of the impeller and the intake, together with an interpretation of the labels. The head H processed by the turbine refers to the medium height of the inlet area $A = L \cdot B$. The “channel width” B is parallel to the axis of the impeller, “transversal dimension” S, as corresponding to a given radius, refers to a plane perpendicular to the impeller axis (in the case of the channel guiding the flow to the impeller, this dimension is the diameter of the circle tangential to the channel walls, while L is the “circumferential length” along the circumference of the impeller. With the approximation mentioned in the introduction, the through-flow can be described with an equally-positioned streamline in the plane of the individual transversal dimensions. In case of a unit channel width, the volume flow is approximately the product of the velocity along the central streamline and the transversal dimension.

By expressing the volume flow that corresponds the turbine's rate of flow Q, the formulae to calculate the intake area A and the channel width B can be obtained:

$$\begin{array}{l} Q = A \cdot c_1 \cdot sin(\alpha_1) \\ A = L \cdot B = R_I \cdot \alpha_1^{\circ} \cdot B \cdot \frac{\pi}{180} \end{array} \Rightarrow \begin{array}{l} A = \frac{1}{sin(\alpha_1) \cdot \sqrt{2gH}} \cdot Q \\ B = \frac{1}{R_I \cdot \varepsilon^{\circ} \cdot \frac{\pi}{180}} \cdot A \end{array} \tag{1}$$

In Figure 2, the left diagram shows the values of the coefficient of the rate of flow *Q (m^3/sec)* according to (1); the intake area is expressed in square meters *(A (m^2))*. The right diagram of Figure 2 shows the values of the coefficient of the intake area *A (m^2)* according to (1); the channel width is expressed in meters *(B (m))*.

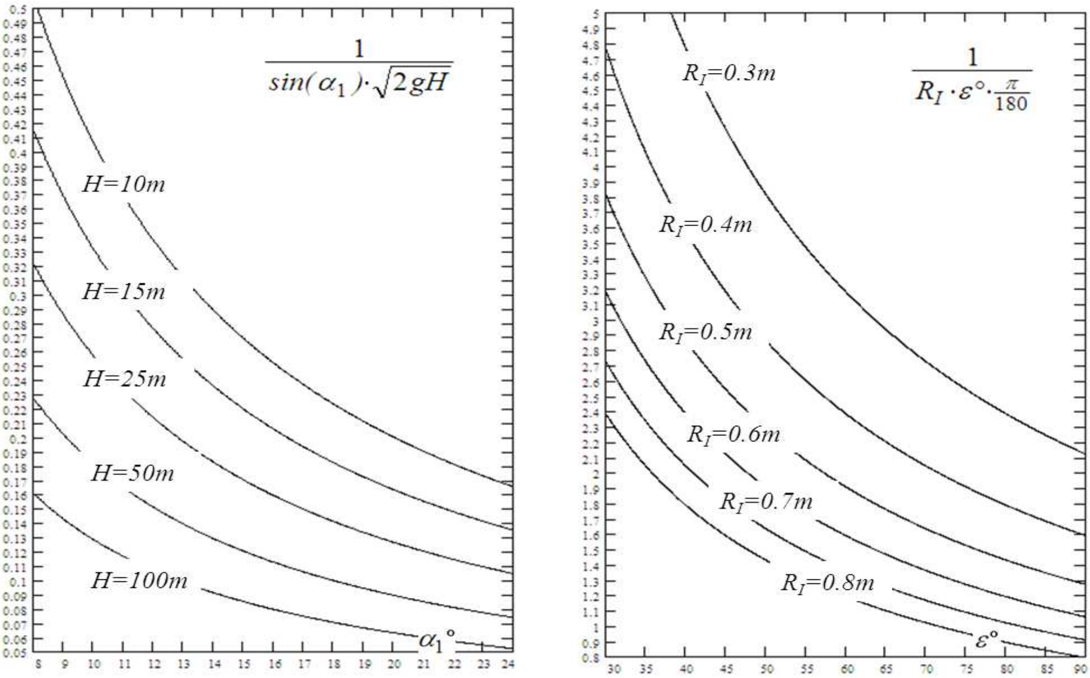

Figure 2

The operating state of the turbine is described by the relation of circumferential speed ωR_I and meridian speed c_{1m} , $\psi = \omega R_I / c_{1m} = u_1 / c_{1m}$. The numerator and the denominator of the parameter of the operating state (ψ) contain data independent of each other. However, these data have a significant influence on the expected shock loss at the inlet and the exit loss. According to out previous publication [3], the maximum value of the shock head loss can be estimated with (2). The amount of the loss varies along the length *L* of the circumference. We assume that an impact-free on-flow occurs at *L/2* on the circumference.

$$h_{shock} = \frac{(\vec{w} - \vec{w}_{ideal})^2}{2g} \Rightarrow \frac{\left[|\vec{u}_1| \cdot sin(\frac{\varepsilon}{2}) \right]^2}{2g} = \frac{\left[u_1 \cdot sin(\frac{\varepsilon}{2}) \right]^2}{2g} \tag{2}$$

In case of an angular momentum free exit, the exit head loss is [4]:

$$h_{outlet} = \frac{{c_{1m}}^2}{2g} \Rightarrow \frac{[2 \cdot u_1 \cdot tg(\alpha_1)]^2}{2g} \tag{3}$$

The diagram of head losses is indicated in Figure 3. The shock loss can be estimated more accurately only when the exact characteristics of the inlet are known. However, the trend shown by Figure 3 applies here, too, i.e. keeping the angle ε as small as possible will help limiting shock loss. A *practical precondition* to an efficient operation is minimizing shock loss at the inlet along the whole circumference of the impeller involved, *L*. This can be obtained only by designing the on-flow channel and the impeller in a coordinated way [2], [3]. A smaller ε angle will decrease interference inside the impeller. This phenomenon can be easily illustrated as the interference of the central streamlines of flow entering at shorter imaginary sections along mantle length L. According to (2) and (3), it is also important to limit circumferential velocity and inlet angle, which of course affects all key dimensions of the impeller.

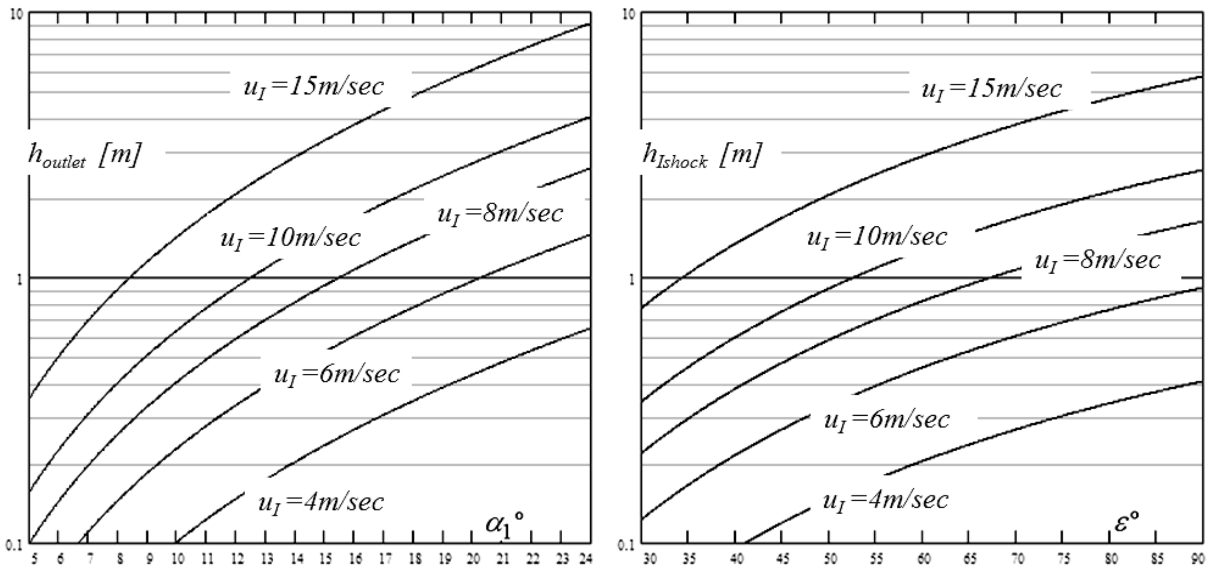

Figure 3

The operating state ψ can be expressed using the ratio of absolute intake velocity and circumferential velocity, u_1 / c_1 ($c_{1m}=c_1 \sin(\alpha_1)$):

$$\psi = \frac{1}{\sin(\alpha_1)} \cdot \frac{u_1}{c_1} \tag{4}$$

Figure 4 shows (4) graphically, with the ratio of absolute intake velocity and circumferential velocity, u_1 / c_1, as parameter. α_1 values are given in degrees. This diagram clearly shows the interdependence of the basic design data (absolute velocity and circumferential velocity at the entry) and the velocity ratio ψ that characterizes the operating state of the turbine and gives information on the direction of the intake velocity. Therefore this diagram simplifies tuning the turbine's operating state and the impeller's dimensions to each other.

The analysis that leads to an optimum compromise should of course take into account many other aspects that are not dealt with in this publication as well (eg. statics, economic etc. aspects).

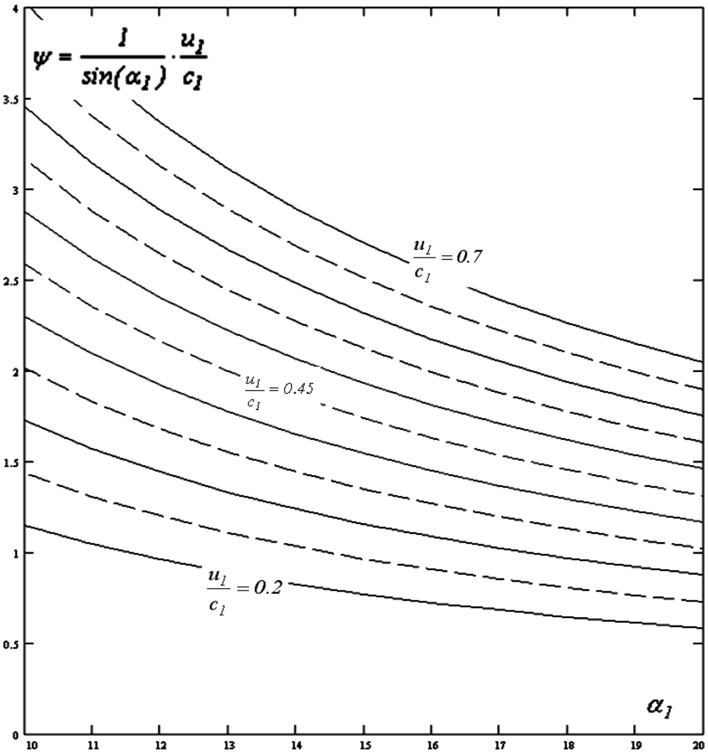

Figure 4

By selecting intake angle α_1, outer impeller radius R_I and angle ε with an optimum compromise, shock and exit losses can be limited for the available water quantity and head. The resulting data system is just a first approximation of the geometrical data that apply along the circumference of the impeller and of the operating state ψ – the data may have to be modified on the basis of the results of the further steps.

3. THE DATA SYSTEM REQUIRED TO OBTAIN AN ANGULAR MOMENTUM FREE OUTLET FLOW IN A CROSS-FLOW TURBINE

The geometrical data inside the impeller (the radius ratio $r=R_{II}/R_I$ and the intake blade angle β_1) depend on the data already fixed along the circumference on the one hand, and their values should be so selected on the other, that the outlet be angular momentum free. In case of an angular momentum free outlet, the flow that exits the impeller is radial. In case of defined dimensions and operating state, the position of the central streamline can be calculated in the impeller [1]. In Figure 1, we illustrated the designations in the picture of the streamline as calculated by Formula [1]. In Figure 1, the flow is radial at the point **e** on radius R_0, where the radius is tangential to the streamline. In the position pictured, the velocity has a circumferential component, while, in case of an angular momentum free exit, the point **e** is located on the circumference, i.e. $R_0 = R_I$ [4], [5].

Our previous publications [4], [5], [6], [7] detail how to calculate the data system valid in the special case of an angular momentum free exit. The formulae are too complex, which is why the relations of the data are only shown in diagrams here (Figure 4 [7]). Figure 4 summarizes the data set that results in an angular momentum free exit in case of the data dealt with by the one-dimensional model. The bordered diagram illustrates the data system in the most compact way. By expanding the data relations, a diagram system suitable for practical use can be assembled, which is also shown in Figure 4. According to Figure 4, assuming an angular momentum free situation, the other data can be calculated from operating state index ψ and radius ratio $r=R_{II}/R_I$, given the inlet blade angle β_1 or the inlet flow angle α_1. The radius ratio required to obtain an angular momentum free exit is related to the operating state via the inlet flow angle α_1.

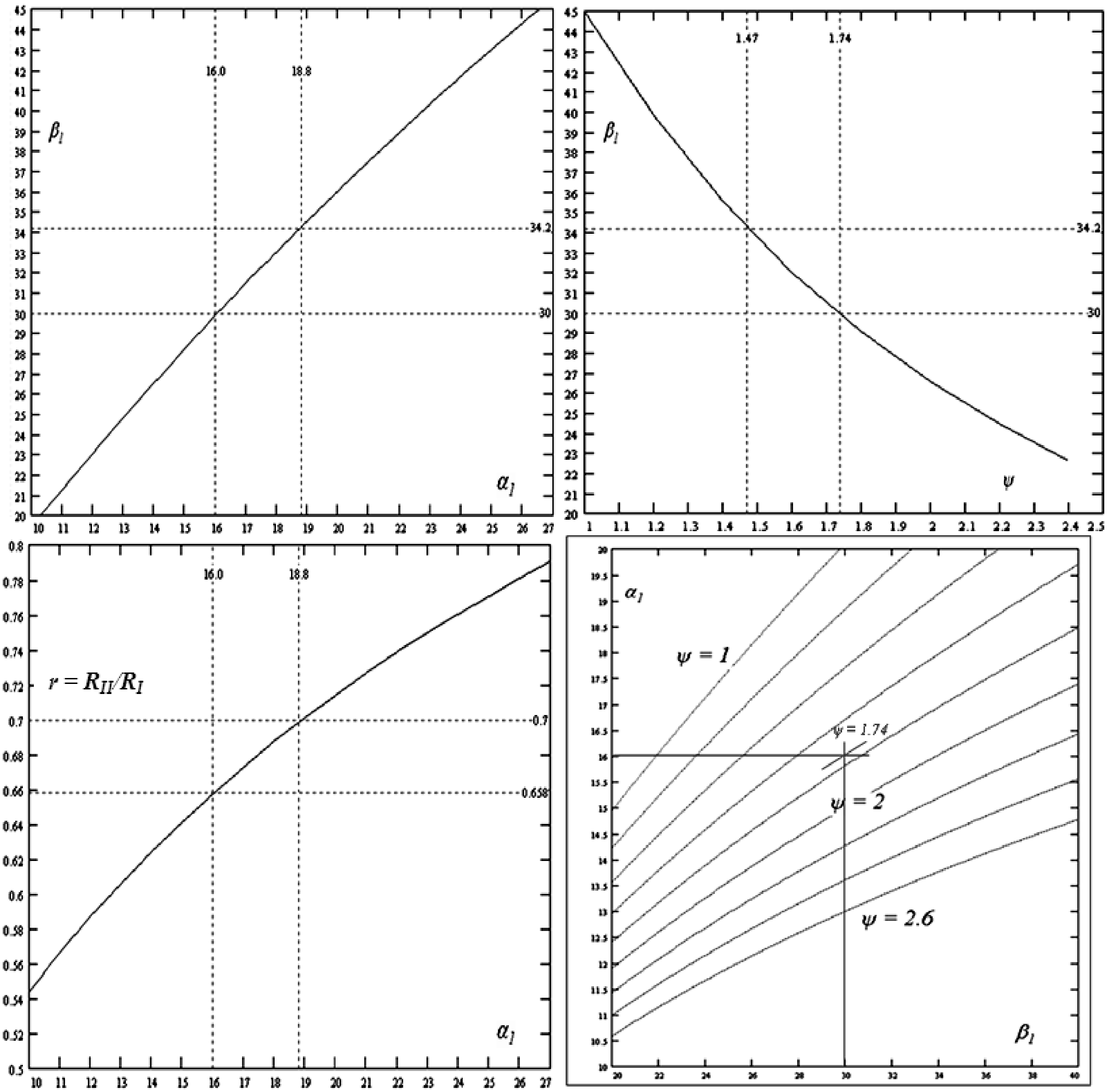

Figure 4

In Figure 4, the dotted guiding lines allow for reading off 2 connected data systems as examples. One of them connects the numerical values of a data set that can be treated as a “rule of thumb”. These are: intake angle $\alpha_1 = 16°$ intake blade angle β_1

$=30°$ and a radius ratio $r = 2/3$. Moreover, the diagrams also show the operating state that corresponds to these data (in the example: $\psi = 1.74$). In this way, the geometrical data system that results in an angular momentum free situation and the data set on the circumference that describes the operating state according to Point 2 can be connected through the data which are common in the two data systems.

4. LIMITING SHOCK LOSS IN THE FLOW ACROSS THE NON-BLADED AREA OF THE IMPELLER OF A CROSS-FLOW TURBINE

As mentioned, assuming given dimensions and operating states, the location of the central streamline in the impeller can be calculated as shown in our previous publication [1] (Figure 1). On this basis, the outlet angle α_i of a flow exiting at a velocity of c_i along the inner mantle of the impeller can be calculated, and, given the transversal dimension of the on-flow channel S_1 at the point **A** on Figure 1, the transversal dimension S_i of the through-flow jet between points **B** and **C** of the impeller using the formula (5). For the calculation, we assume that the relative flow of a velocity of w_i enters inside of the blading in a radial way, i.e. w_i and u_i are perpendicular to each other.

$$c_1 \cdot S_1 = c_i \cdot S_i$$

$$u_1 \cdot \frac{R_{II}}{R_I} = u_i = c_i \cdot cos(\alpha_i) \qquad \Rightarrow \qquad S_i = \frac{c_1 \cdot S_1}{\frac{u_1}{cos(\alpha_i)} \cdot \frac{R_{II}}{R_I}} = \frac{c_1 \cdot S_1 \cdot cos(\alpha_i)}{u_1 \cdot r} \tag{5}$$

The jet of a width of S_i can be drawn, using the streamline considered to be straight as centerline. The contour corresponding to the axis of the impeller can also be drawn, and therefore it can be checked whether the flow avoids the impeller axis and what is the length of the jet crossing the inside contour of the blading. If required, the design/analysis procedure must be repeated in order to limit the impact loss. Our publications [4], [5] showed that an angular momentum free axis can be obtained by influencing the position of the central streamline (in case of an angular momentum free exit, the tangent to the streamline is radial), therefore the repeated design/analysis affects all design parameters.

5. GEOMETRY OF THE IMPELLER BLADES

In Figure 1, the circular-shaped blade is illustrated as the EA curve with center **K** and radius R. The tangent to the blade runs radially at point E. The radius R can be calculated using (6) [1].

$$R = \frac{R_I^2 - R_{II}^2}{2R_I \cos \beta_1} \tag{6}$$

The centers **K** of the circular-shaped blades are located along the circle of a radius of R_{III} to be calculated using (7).

$$R_{III} = \sqrt{(R_I - R \cdot cos\,\beta_1)^2 + (R \cdot sin\,\beta_1)^2} \tag{7}$$

The positioning of the blades in the blading is much easier when the central angle γ that belongs to the circular arc sections that make up the blade (Figure 1). The intersection points of the legs and the circles with the radiuses R_I and R_{II} set up the end-points of circular arc-shaped (radius R) blades in a way that the blade angle corresponding to the radius R_I is β_1 and the tangent to the blade curve at the radius R_{II} is radial. The relationship between the intake blade angle β_1, the radiuses R_I and R_I, and the central angle γ is given by (8). Using (6) and (8), geometrical data regarding the blade curve and the position of the blade within the blading can be calculated. In case of the blading, it is not advisable to use diagrams despite the complexity of the expressions, in order to keep the accuracy. (7) and (8) can also be used for checking: the intersection point of the circular arcs of radius R, running from the end-points of the blade curve as calculated above, will mark the center **K** of the circular arc, which is located on the circle of a radius R_{III}, while using the explicit expression of β_1, the susceptibility of the intake angle to the dimensional manufacturing errors can be examined.

$$\beta_1 = arctg\left[\frac{1}{tan\,\gamma}\left[\left(R_I - \frac{R_{II}}{cos\,\gamma}\right)\frac{2R_I}{R_I^2 - R_{II}^2} - 1\right]\right]$$
$$\gamma = -2\cdot arctg\left[\frac{(R_I - R_{II})\cdot(sin(\beta_1) - 1)}{(R_I + R_{II})\cdot cos(\beta_1)}\right] \qquad (8)$$

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7. IRODALOM

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